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RELIABILITY-BASED SCATTER FACTORS
Volume I: Theoretical and Empirical Results

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A definition of scatter factor is introduced that is rational and at the same time can directly be related to the reality of aircraft design and certification as well as of the full-scale and also coupon fatigue test of structural elements or components. Specifically, the scatter factor is defined as the ratio of the MLE (maximum likelihood estimate) of the scale parameter of the two-parameter Weibull distribution assumedly describing the life distribution of structural elements or components, to the "time to first failure" among a

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fleet of nominally identical elements or components subjected also to nominally identical operating conditions. Freudenthal has used the same definition of the scatter factor, however, under much simplified conditions: He assumes that the shape parameter of the Weibull distribution is known. This assumption is mathematically highly convenient since it permits the derivation of the distribution of the scatter factor in closed form and independent of the unknown scale parameter. Unfortunately, however, such an assumption is inconsistent with the reality where the Weibull shape parameter easily ranges from 2.0 to 10.0 reflecting the fact that structural elements or components suffer from a variety of sources of randomness in fatigue strength; not only from the probabilistic variation of the material property but also from the statistical variation in workmanship associated with, for example, drilling rivet holes in the process of airframe fabrications. The mathematical difficulty, however, multiplies when the Weibull shape and scale parameters are both assumed to be unknown. Procedures involving Monte Carlo techniques have been established to evaluate the scatter factor under these conditions, using the maximum likelihood estimates of the parameters. The fleet reliability can then be estimated on the basis of the scatter factor thus evaluated. The effect of the sample size to be used in the fatigue test, of the fleet size and of the reliability level on the accuracy of such estimation has also been discussed.

FOREWORD

The research work reported herein was conducted at Modern Analysis Inc., Ridgewood, New Jersey, for the Air Force Flight Dynamics Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, under contract F33615-77-C-3055, project 2304N106, Reliability-Based Scatter Factors, with Dr. H. Leon Harter (AFFDL/FBRD) acting as project engineer.

The research was performed by Dr. M. Shinozuka of Modern Analysis Inc. as principal investigator. The computer programs were developed by Mr. D. Li of Columbia University. Work began March 1977 and was completed March 1978. The final report in two volumes was submitted in March, 1978.

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INTRODUCTION

In a recent study dealing with reliability-based criteria for design, inspection and fleet management of USAF aircraft[1*], an effort has been made to incorporate into the reliability evaluation scheme the direct or indirect effect of material selection, geometrical configuration (in terms of either fail safe or slow crack growth model), mission spectra, inspection procedures, proof load test, design practice and analysis method. This study has not only provided an adequate analytical framework for the current effort toward implementation of reliability-based criteria but also identified a number of difficulties which should be alleviated, if not totally removed, by means of a continued study before reliability-based criteria can be accepted and implemented with sufficient engineering confidence. Among the items on which the study recommended a continued investigation is the scatter factor which has been a part of the conventional design procedure and at the same time is closely related to the reliability of aircraft structures as recently demonstrated by Freudenthal[2].

The general framework for the development of reliability-based criteria for aircraft must emerge as a compromise among the applicability of rigorous analytical procedures, the

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* Numerals indicate references listed on p. 72.

availability of pertinent data and the requirements of ready implementation of such criteria. These requirements therefore should provide a format that makes it reasonably simple to translate design procedures and design-decision processes currently in use in the U. S. Air Force and in the airframe industry into formally not too dissimilar procedures and processes which reflect, however, the new probabilistic concept of engineering reality. The scatter factor indeed has such dual characteristics. Also, the scatter factor, being probably the only quantity based on the full-scale test, plays a crucial role in the certification and reliability demonstration procedures.

In the study performed here, introduced is a definition of the scatter factor that is rational and at the same time can directly be related to the reality of aircraft design and certification as well as of the full-scale and also coupon fatigue tests of structural elements or components. Specifically, the scatter factor will be defined as the ratio of the MLE (maximum likelihood estimate) of the scale parameter of the two-parameter Weibull distribution assumedly describing the life distribution of structural elements or components, to the "time to first failure" among a fleet of nominally identical elements or components subjected also to nominally identical operating conditions. Freudenthal has used in Ref. 2 the same definition of the scatter factor, however, under much simplified conditions: He assumes that the shape parameter of the Weibull distribution

is known. This assumption is mathematically highly expedient since it permits the derivation of the distribution of the scatter factor in closed form and independent of the unknown scale parameter as demonstrated in Ref. 2. Unfortunately, however, such an assumption is inconsistent with reality where the Weibull shape parameter easily ranges from 2.0 to 10.0, reflecting the fact that structural elements or components suffer from a variety of sources of randomness in fatigue strength; not only from the probabilistic variation of the material property but also from the statistical variation in workmanship associated with, for example, drilling rivet holes in the process of airframe fabrications.

The mathematical difficulty multiplies when the Weibull shape and scale parameters are both assumed to be unknown. The proposed Monte Carlo simulation approach can overcome the difficulty and produce results amenable to practical applications.

SECTION II

CRITICAL REVIEW OF FREUDENTHAL'S SCATTER FACTOR

Recently, Freudenthal published a paper[2] proposing a statistical interpretation of the scatter factor to be used in the reliability assessment of aircraft structures. The essence of his paper is briefly described below.

Consider a two-parameter Weibull distribution for the life t of a structure (or a structural component):

$$F_0(t) = 1 - \exp[-(\frac{t}{\beta})^{\alpha}] \quad (1)$$

where it is assumed that the shape parameter α is known while the scale parameter β is unknown. The maximum likelihood estimate (MLE) of β is given by

$$\hat{\beta} = [\frac{1}{n} \sum_{i=1}^{n} t_{0i}^{\alpha}]^{1/\alpha} \quad (2)$$

with $t_{01}, t_{02}, \ldots.., t_{0n}$ indicating a sample of size n taken from the distribution given by Eq. 1.

Let t_1 represent the time to first failure (minimum life) in a fleet of m aircraft (fleet size = m). Then, the distribution function of t_1 is given by

$$F_1(t_1) = 1 - \exp[-(\frac{t_1}{\beta_1})^{\alpha}] \quad (3)$$

where

$$\beta_1 = \beta/m^{1/\alpha} \tag{4}$$

Freudenthal defines the scatter factor S as

$$S = \hat{\beta}/t_1 \tag{5}$$

and shows that the distribution of S is given by

$$F_S(s) = \left[\frac{s^\alpha}{(m/n) + s^\alpha} \right]^n \tag{6}$$

In deriving Eq. 6, the fact has been used that $2n(\hat{\beta}/\beta)^\alpha$ is distributed as χ^2 with 2n degrees of freedom and hence the density function of $\hat{\beta}$ is given by

$$f_2(\hat{\beta}) = \frac{n^n}{\Gamma(n)} \cdot \frac{\alpha}{\beta} \cdot \left(\frac{\hat{\beta}}{\beta}\right)^{\alpha n-1} \exp[-n(\hat{\beta}/\beta)^\alpha] \tag{7}$$

It then follows that the density function of $z = \hat{\beta}/\beta$ is

$$g_2(z) = \frac{n^n}{\Gamma(n)} \cdot \alpha \cdot z^{\alpha n-1} \exp[-nz^\alpha] \tag{8}$$

Consider, at this point, the following transformations:

$$v = (\hat{\beta}/\beta)^\alpha = \frac{1}{n} \sum_{i=1}^{n} (t_{0i}/\beta)^\alpha \tag{9}$$

and

$$w = (t_1/\beta)^\alpha \tag{10}$$

Note that $(t_{0i}/\beta)^\alpha$ is exponentially distributed with unit mean

value (indeed, Eq. 7 follows from this fact) and that $(t_1/\beta)^\alpha$ is also exponentially distributed but with mean value equal to 1/m. Therefore, it is easy to generate sample pairs of v and w by Monte Carlo techniques and at the same time compute the sample values of the ratio v/w therefrom. This ratio can be written from Eqs. 9 and 10 as

$$v/w = (\hat{\beta}/t_1)^\alpha \tag{11}$$

and hence

$$S = \hat{\beta}/t_1 = (v/w)^{1/\alpha} \tag{12}$$

Since α has been assumed to be known, sample values of scatter factor S can be generated by means of Eq. 12. Indeed, the use of such a Monte Carlo simulation with the size of the (simulated) sample as large as 999 has resulted in a satisfactory result in producing the distribution of scatter factor S for the case of $m = 3$, $n = 1$ and $\alpha = 4.0$ as shown in Fig. 1.

Although the agreement just observed between the simulation and the theory is for a particular set of parameter values, i.e., $m = 3$, $n = 1$, $\alpha = 4.0$, it is expected that the sample size of the order of magnitude of 1,000 will be sufficiently large for the type of simulations to be performed later in which no theoretical distribution is available for comparison purposes.

Define now t_1^* as the service life specified for the fleet or equivalently as the specified minimum life in the fleet. Define also the fleet reliability R as

$$R = P\{t_1 \geq t_1^*\} \tag{13}$$

which indicates the probability that the time to first failure in the fleet will be larger than or equal to the specified value. In Ref. 2, however, Freudenthal implied that the probability

$$R' = P\{S \leq \hat{\beta}_0/t_1^*\} \tag{14}$$

should be used for the fleet reliability, where $\hat{\beta}_0$ now indicates a realization of random variable $\hat{\beta}$. With the aid of Eqs. 3 and 4, the fleet reliability can be written as

$$R = \exp[\,-\,m(t_1^*/\beta)^{\alpha}] \tag{15}$$

while R' in Eq. 14, with the aid of Eq. 6, becomes

$$R' = \left[\frac{(\hat{\beta}_0/t_1^*)^{\alpha}}{m/n + (\hat{\beta}_0/t_1^*)^{\alpha}}\right]^n \tag{16}$$

Obviously R and R' are not identical. The difference can be more clearly demonstrated by rewriting Eqs. 13 and 14 respectively as

$$R = \iint_{D_1 U D_2} f_1(t_1) f_2(\hat{\beta}) dt_1 d\hat{\beta} \tag{17}$$

$$R' = \iint_{D_1 U D_3} f_1(t_1) f_2(\hat{\beta}) dt_1 d\hat{\beta} \tag{18}$$

where the domains of integration D_1UD_2 (D_1 union D_2) and D_1UD_3

are shown in Fig. 2. The domain $D_1 \cup D_3$ in Eq. 18 can be obtained by adding D_3 to and subtracting D_2 from the correct domain of integration $D_1 \cup D_2$. Note that D_3 represents a conservative addition and D_2 an unconservative subtraction.

To demonstrate that R' may be used as an approximation to R, consider the expected value E[R'] of R';

$$E[R'] = \int_0^\infty R' f_2(\hat{\beta}) d\hat{\beta} = \int_0^\infty R' g_2(z) dz \tag{19}$$

where R' and $f_2(\hat{\beta})$ in the integrand of the second member are to be replaced by the right hand sides of Eqs. 16 and 7 respectively. The integration then depends on t_1^*, β and m. These quantities are related, however, in the following fashion through Eq. 15.

$$(t_1^*/\beta)^\alpha = -(\ln R)/m \tag{20}$$

With the aid of Eq. 20, t_1^* and m in Eq. 16 (with $\hat{\beta}_0$ being replaced by $\hat{\beta}$) are eliminated and furthermore with the aid of the definition $z = \hat{\beta}/\beta$, Eq. 16 becomes

$$R' = \left[\frac{(\hat{\beta}/\beta)^\alpha}{(\hat{\beta}/\beta)^\alpha - (\ln R)/n}\right]^n = \left[\frac{z^\alpha}{z^\alpha - (\ln R)/n}\right]^n \tag{21}$$

Substituting Eqs. 8 and 21 into the third member of Eq. 19, one obtains

$$E[R'] = \int_0^\infty \left[\frac{nz^\alpha}{nz^\alpha - \ell n\, R}\right]^n \frac{\alpha n^n}{\Gamma(n)} z^{\alpha n-1} \exp[-nz^\alpha]\,dz \qquad (22)$$

Similarly,

$$E[(R')^2] = \int_0^\infty \left[\frac{nz^\alpha}{nz^\alpha - \ell n\, R}\right]^{2n} \frac{\alpha n^n}{\Gamma(n)} z^{\alpha n-1} \exp[-nz^\alpha]\,dz \qquad (23)$$

The standard deviation $\sigma_{R'}$ of R' can easily be obtained from Eqs. 22 and 23.

The integrals in Eqs. 22 and 23, now independent of m, are evaluated numerically for α = 0.5, 1.0, 2.0, ..., 10.0, R = 0.5, 0.6, ..., 0.9, 0.99, 0.999, 0.9999 and n = 1, 2, ..., 10. The ratios E[R']/R for these values of R are then plotted as functions of α in Figs. 3 - 12 for n = 1, 2, ..., 10, respectively. Also, the coefficients of variation $V_{R'} = \sigma_{R'}/E[R']$ of R' are plotted as functions of α for the same values of R and n in Figs. 13 - 22. One observes from these results the following general trend.

1) As α increases, so does the ratio E[R']/R. However, except for a sharp increase observed between α = 0.5 and 1.0, the rate of increase is small. In fact, the ratio is almost constant for those values of $\alpha \geq 2.0$ which include a practical range of α between 2.0 and 5.0.
2) For the same value of R, the ratio is closer to unity for a larger value of n.
3) The ratio is not necessarily larger for a larger value

of R, although numerical results indicate that E[R'] monotonically increases as a function of R with all other parameters kept constant.

4) As α increases, the coefficient of variation $V_{R'}$ of R' decreases. Except for a sharp decrease observed between $\alpha = 0.5$ and 2.0, however, the rate of decrease is small. The coefficient is almost constant for $\alpha \geqq 2.0$.

5) For the same value of R, the coefficients are smaller when n is larger.

6) The coefficients are smaller for a larger value of R.

For example, the ratio E[R']/R is nearly equal to 0.950 for R = 0.5 and n = 1 (Fig. 3). This may give an impression that the approximation is acceptable. However, the fact that the corresponding coefficient of variation $V_{R'}$ is as large as 0.50 (Fig. 13), proves that the impression is unsubstantiated. As n increases to 5 and to 10 (while R = 0.5 is kept constant), the ratio E[R']/R increases to 0.970 and to 0.982 (Figs. 7 and 12) and at the same time, the coefficient $V_{R'}$ decreases to 0.30 and to 0.22 respectively (Figs. 17 and 22), thus making the approximation more reasonable. For R = 0.9, however, the approximation appears to be reasonable since then the ratios are 0.88, 0.98, 0.99 and the coefficients are 0.27, 0.07 and 0.05 respectively for n = 1, 5, and 10. Tables 1 - 3 summarize such observations for selected values of R, α and n. The last column in Tables 1 - 3 lists the value of Freudenthal's scatter factor corresponding to E[R'] for different values of m.

SECTION III

STATISTICAL SCATTER FACTOR WITH UNKNOWN SHAPE AND SCALE PARAMETERS

It is well known that, when the shape and scale parameters of the two parameter Weibull distribution given in Eq. 1 are both unknown, the MLE $\hat{\alpha}$ of α and MLE $\check{\beta}$ of β are obtained from the following simultaneous equations:

$$n = \sum_{i=1}^{n} (t_{0i}/\check{\beta})^{\hat{\alpha}} \tag{24}$$

$$n = \sum_{i=1}^{n} [(t_{0i}/\check{\beta})^{\hat{\alpha}} - 1] \ln(t_{0i}/\check{\beta})^{\hat{\alpha}} \tag{25}$$

where, as before, $t_{01}, t_{02}, \ldots, t_{0n}$ indicate a sample of size n taken from the Weibull distribution. In this case, the statistical scatter factor Q is introduced in the following form as a natural extension of S given in Eq. 5.

$$Q = \check{\beta}/t_1 \tag{26}$$

Defining $y(i)$, u and v_0 as

$$y(i) = (t_{0i}/\beta)^{\alpha} \tag{27}$$

$$u = \hat{\alpha}/\alpha \tag{28}$$

and

$$v_0 = (\check{\beta}/\beta)^{\alpha} \tag{29}$$

one can rewrite Eqs. 24 and 25 respectively in the following forms.

$$v_0 = [\sum_{i=1}^{n} y^u(i)/n]^{1/u} \tag{30}$$

$$\phi(u) = \sum_{i=1}^{n} y^u(i)\ln y(i)/\sum_{i=1}^{n} y^u(i) - 1/u - \sum_{i=1}^{n} \ln y(i)/n = 0 \tag{31}$$

Since y(i) is exponentially distributed with mean value equal to unity, the joint distribution of u and v_0 can be obtained through Eqs. 30 and 31 by the Monte Carlo simulation technique. All that one is required to do is to generate y(i) (i = 1, 2, ..., n), use them in Eqs. 30 and 31, solve these two equations for u and v_0, and repeat the process N times. This will produce a simulated sample of u and v_0 pairs of size N. Independently of the simulation of u and v_0, generate again a (simulated) sample of w (Eq. 10) of size N.

By means of the Monte Carlo simulation technique just described, Whittaker and Besuner[3] constructed the empirical distribution functions of u* and v_0^* defined as

$$u^* = 1/u \tag{32}$$

$$v_0^* = v_0^{\,u} = (\check{\beta}/\beta)^{\hat{\alpha}} \tag{33}$$

These empirical distributions are simulated, based on samples of size 1,999, and are shown in Figs. 23 and 24 respectively. The comparison of current simulation results with those in Ref. 3 indicates that the accuracy of the current simulation is

generally comparable with that of Ref. 3.

Since

$$Q^{\alpha} = (\check{\beta}/t_1)^{\alpha} = (\check{\beta}/\beta)^{\alpha}/(t_1/\beta)^{\alpha} = v_0/w \tag{34}$$

it follows that

$$Q^* = Q^{\hat{\alpha}} = (v_0/w)^u \tag{35}$$

The last equation indicates that the distribution of Q^* can be constructed with the aid of u, v_0 and w generated above. For the ease of application, however, the distribution of the logarithm Z of Q^*,

$$Z = \log_{10} Q^* = u \log_{10}(v_0/w) \tag{36}$$

is constructed and plotted in Figs. 25 - 29 (respectively for $m = 1,3,5,25,$and 100), where solid circles indicate the values of the empirical distribution function of Z based on samples of size 1,999. Solid curves are drawn through these circles by interpolation. Similarly, Fig. 30 plots the empirical distribution of Z for various values of m when $n = 3$.

Having thus established the empirical distribution of Z, define the probability R'' as

$$R'' = P\{Q \leqq \check{\beta}/t_1^*\} = P\{Q^{\hat{\alpha}} \leqq (\check{\beta}/t_1^*)^{\hat{\alpha}}\} \tag{37}$$

This probability R" is similar to Freudenthal's fleet reliability R' defined by Eq. 14 and can be expressed, with the aid of Eqs. 35 - 37, as

$$R'' = P\{Z \leqq \log_{10}(\check{\beta}/t_1^*)^{\hat{\alpha}}\} \tag{38}$$

In Eqs. 37 and 38, t_1^* is the specified minimum life, and $\hat{\alpha}$ and $\check{\beta}$ represent realizations of the maximum likelihood estimates of α and β based on a sample of size n taken from Eq. 1; t_{01}, t_{02}, ..., t_{0n}. The empirical distribution of Z, such as is established numerically and graphically, for example, in Fig. 25, can thus be used to find the probability R" given a sample of t.

Since one can show with the aid of Eq. 20 that

$$(\check{\beta}/t_1^*)^{\hat{\alpha}} = \left(\frac{mv_0}{\ln(1/R)}\right)^u \quad , \tag{39}$$

Eq. 38 can be written as

$$R'' = P[Z \leqq u \log_{10}\{\frac{mv_0}{\ln(1/R)}\}] = F_Z[u \log_{10}\{\frac{mv_0}{\ln(1/R)}\}] \tag{40}$$

where $F_Z[\cdot]$ is the (cumulative) distribution function of Z. In the computation that follows, empirical distributions such as those obtained in Figs. 25 - 30 are used in place of $F_Z[\cdot]$. The expected values of $(R'')^k$, where k is a positive integer, can then be evaluated as

$$E[(R'')^k] = \int_0^\infty \int_0^\infty F_Z^k[u \log_{10}\{\frac{mv_0}{\ln(1/R)}\}] \cdot f_{uv_0}(u,v_0)dudv_0 \qquad (41)$$

Obviously, the expected value of R" is obtained as E[R"] and the variance of R" as $Var[R''] = E[(R'')^2] - E^2[R'']$. In Eq. 41, $f_{uv_0}(\cdot, \cdot)$ is the joint density function of u and v_0, which can be evaluated empirically by making use of Eqs. 30 and 31. In fact, the random variables $u^* = 1/u$ and $v_0^* = v_0{}^u$ considered before have joint distribution resulting from the same underlying dependence as for u and v_0 and the empirical distributions shown in Figs. 23 and 24 are their respective marginal distribution functions. The (two-dimensional) histograms of u and v_0 are shown in Figs. 31 - 35 for the cases n = 2, 3, 5, 10 and 20, each on the basis of a simulated sample of N = 1,999 pairs of u and v_0. The empirical density can then be obtained by dividing the frequency in the histogram by 1,999. The sum of the frequency values in each of these histograms may not add up to 1,999 because some the (simulated) observations fell outside of the domain of u and v_0 considered here; u = 0 - 20 and v_0 = 0 - 4.

Making use of the empirical distribution of Z and the empirical density of u and v_0, one can evaluate Eq. 41 for k = 1 and 2 and hence find the mean value and the coefficient of variation $V_{R''}$ of R". The results of computation are shown in Figs. 36 - 40 with R as abscissa for the cases n = 2, 3, 5, 10 and 20, and also in Figs. 41 - 45 with E[R"] as abscissa. Similar to Tables 1 - 3, Tables 4 - 8 summarize these results

for selected values of R, m and n. In Tables 4 -8, however, the values of Q* corresponding to the fleet reliability R = E[R"] are listed in place of S.

The following observations can be made on the basis of these results.

1) The ratio E[R"]/R increases as n and R increase but decreases as m increases. For the ranges of n, R and m examined here, the smallest (worst) is 0.655 (n = 2, R = 0.95 and m = 100) while the largest (best) is 0.995 (n = 20, R = 0.999 and m = 1). This indicates that E[R"] and R are essentially of the same order of magnitude in these ranges of n, R and m.

2) The coefficient of variation $V_{R"}$ decreases as n and R increase while it increases as m increases.

3) On the basis of the observations (1) and (2) above, one can determine for what combinations of n, R and m the fleet reliability R" based on the scatter factor (computed from Eq. 37) can be used as an approximation in place of the (true) fleet reliability R. For example, if one considers a coefficient of variation $V_{R"}$ less than or equal to 0.20 as acceptable, the fleet size m must be less than or equal to 5 at the reliability level of 0.99 for R" to be acceptable if n = 2 (see Fig. 36) whereas R" may be used in place of R even with the fleet size as large as m = 100 at the same level of fleet reliability of 0.99 if n = 20 (see Fig. 40).

4) Figs. 41 - 45 are particularly useful in practical applications. Realizations $\hat{\alpha}_0$ and $\check{\beta}_0$ of $\hat{\alpha}$ and $\check{\beta}$ respectively on a sample of size n can be used in Eq. 38 to obtain a realization of R''_0 of R''. Actually, this is done by reading the value of R'' from Figs. 25 - 30. Diagrams showing $V_{R''}$ as a function of $E[R'']$ in Figs. 41 - 45 then indicate whether R''_0 just obtained can be used for $E[R'']$ depending on the fleet size m and the sample size n: If $V_{R''}$ corresponding to $E[R''] = R''_0$ is small (say, less than 0.20) for m and n under consideration, $E[R'']$ may indeed be considered equal to R''_0 in approximation. Using this value of $E[R'']$ in the diagrams showing $E[R'']/R$ as a function of $E[R'']$ in Figs. 41 - 45, we can estimate the corresponding value of R. As this process of evaluating R suggests, Figs. 41 - 45 implicitly indicate for what combinations of m, n and the specified value of $V_{R''}$ the probability R''_0 can be used as an approximation to R. If the value of $V_{R''}$ corresponding to R''_0 (replacing $E[R'']$ in Figs. 41 - 45) exceeds the specified value of $V_{R''}$ for a particular set of m and n, then one or any combination of the following should be implemented to make R''_0 a better approximation to R: (a) The fleet size m to be considered should be reduced. This requires a reevaluation of R''_0. (b) The sample size n should be increased. This requires testing of an additional number of structural components and a reevaluation of R''_0 based on that, and (c) the specified minimum life t_1^* should be reduced. This requires a reevaluation of R''_0.

SECTION IV

CONCLUSION

The concept of the statistical scatter factor has been applied to the case where the parent distribution of the fatigue life of aircraft or their components is a two-parameter Weibull with both shape and scale parameters being unknown. Procedures have been established to evaluate the scatter factor in its extended form using the maximum likelihood estimates of the parameters. The fleet reliability can then be estimated on the basis of the scatter factor of extended form thus evaluated. The effect of the sample size to be used in the fatigue test, of the fleet size and of the reliability level on the accuracy of such estimation has also been discussed.

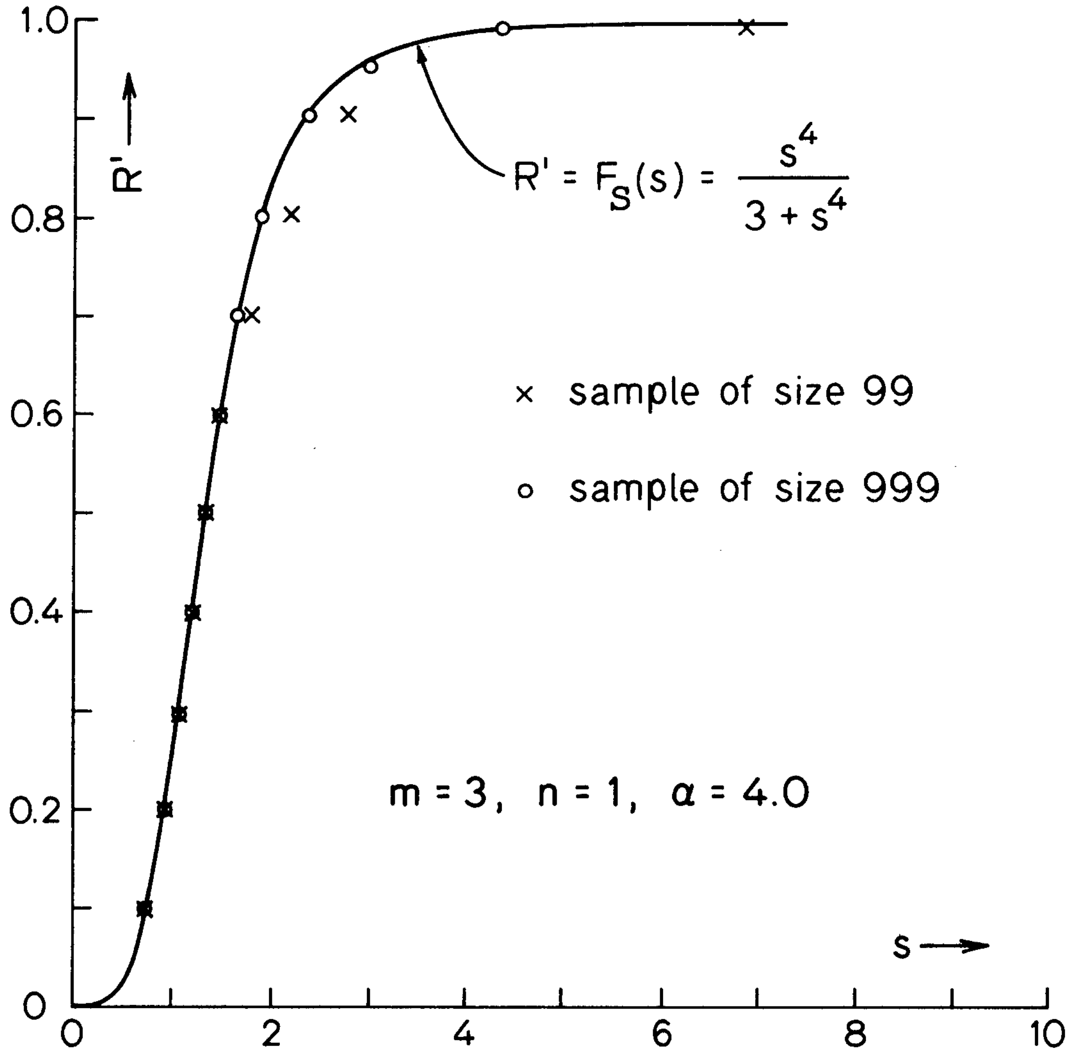

Fig. 1. Distribution function of scatter factor.

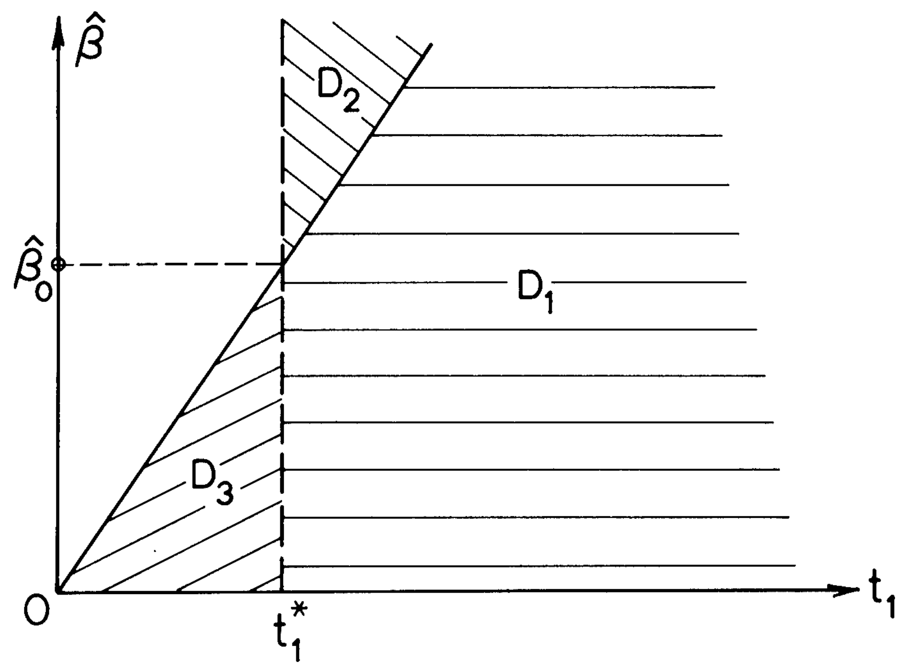

Fig. 2. Domains of integration.

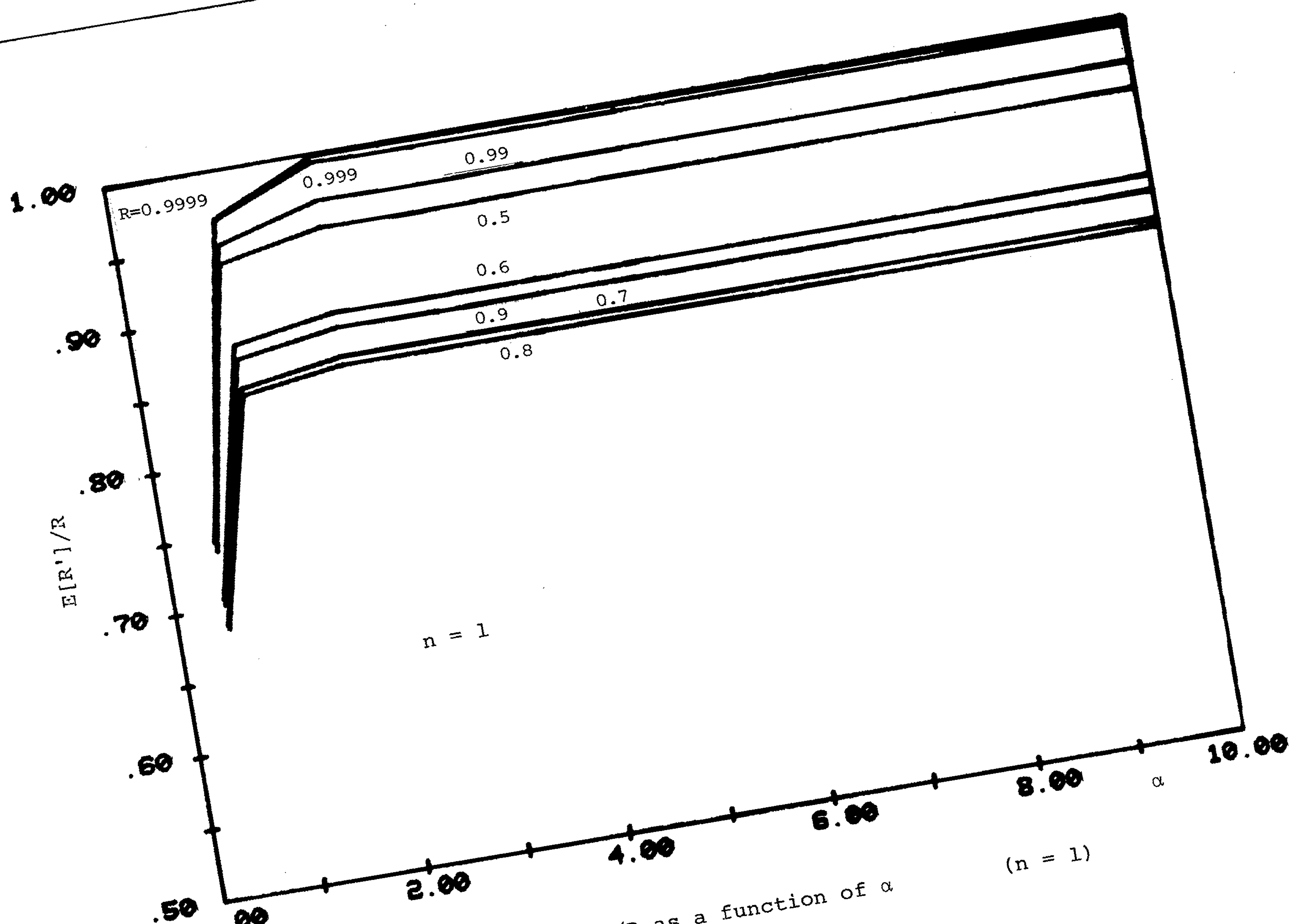

Fig. 3. E[R']/R as a function of α (n = 1)

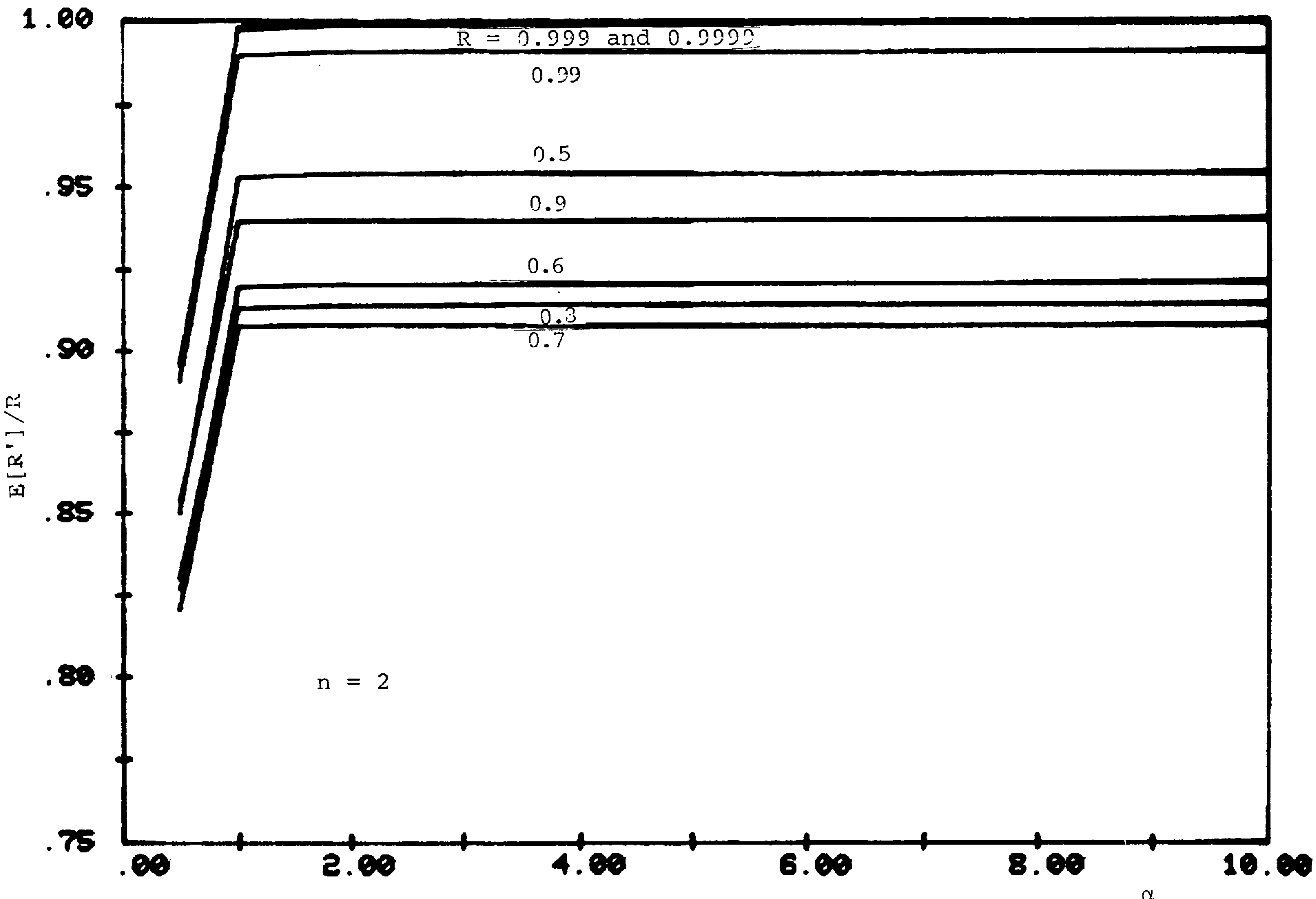

Fig. 4. E[R']/R as a function of α (n = 2)

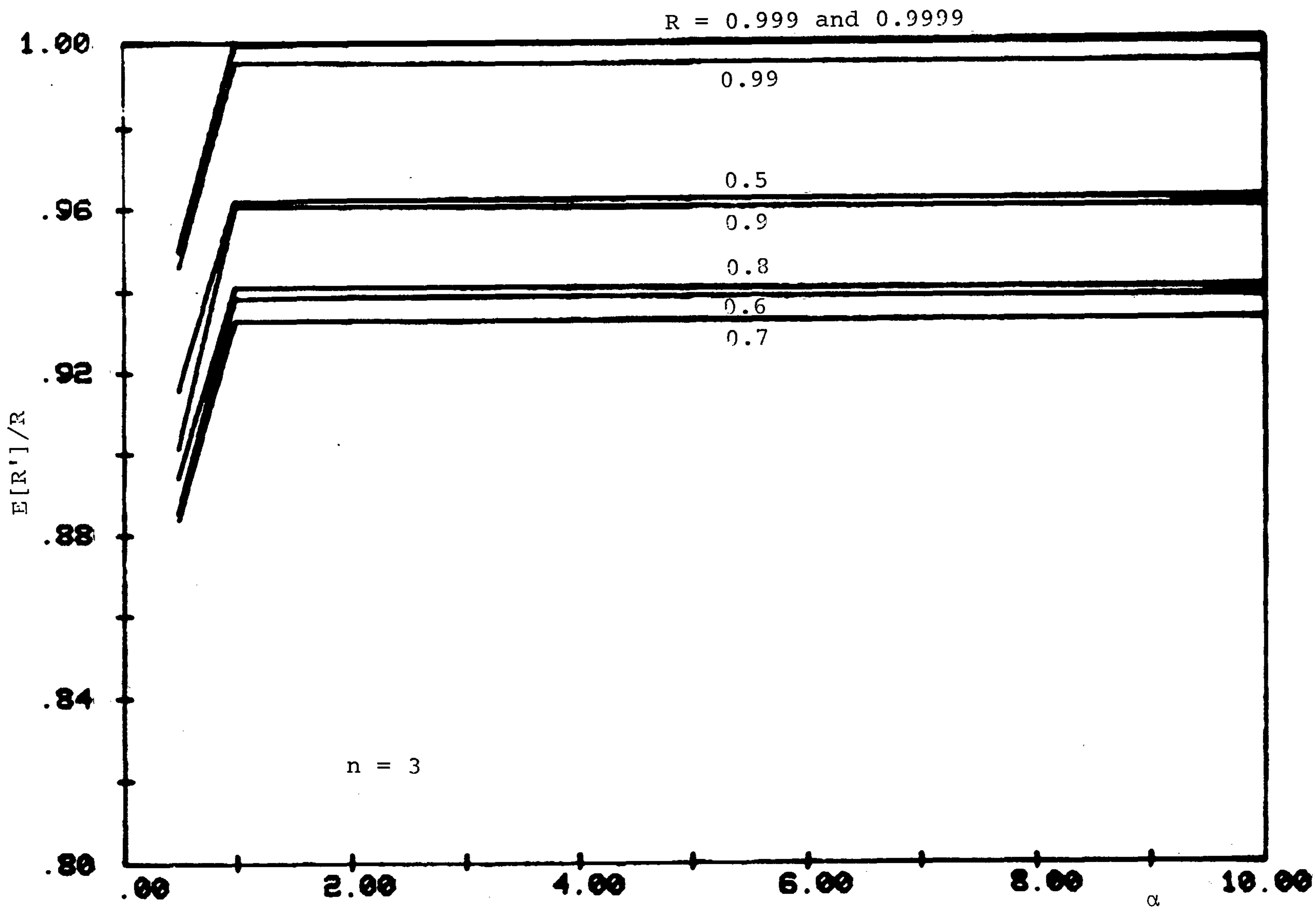

Fig. 5. E[R']/R as a function of α (n = 3)

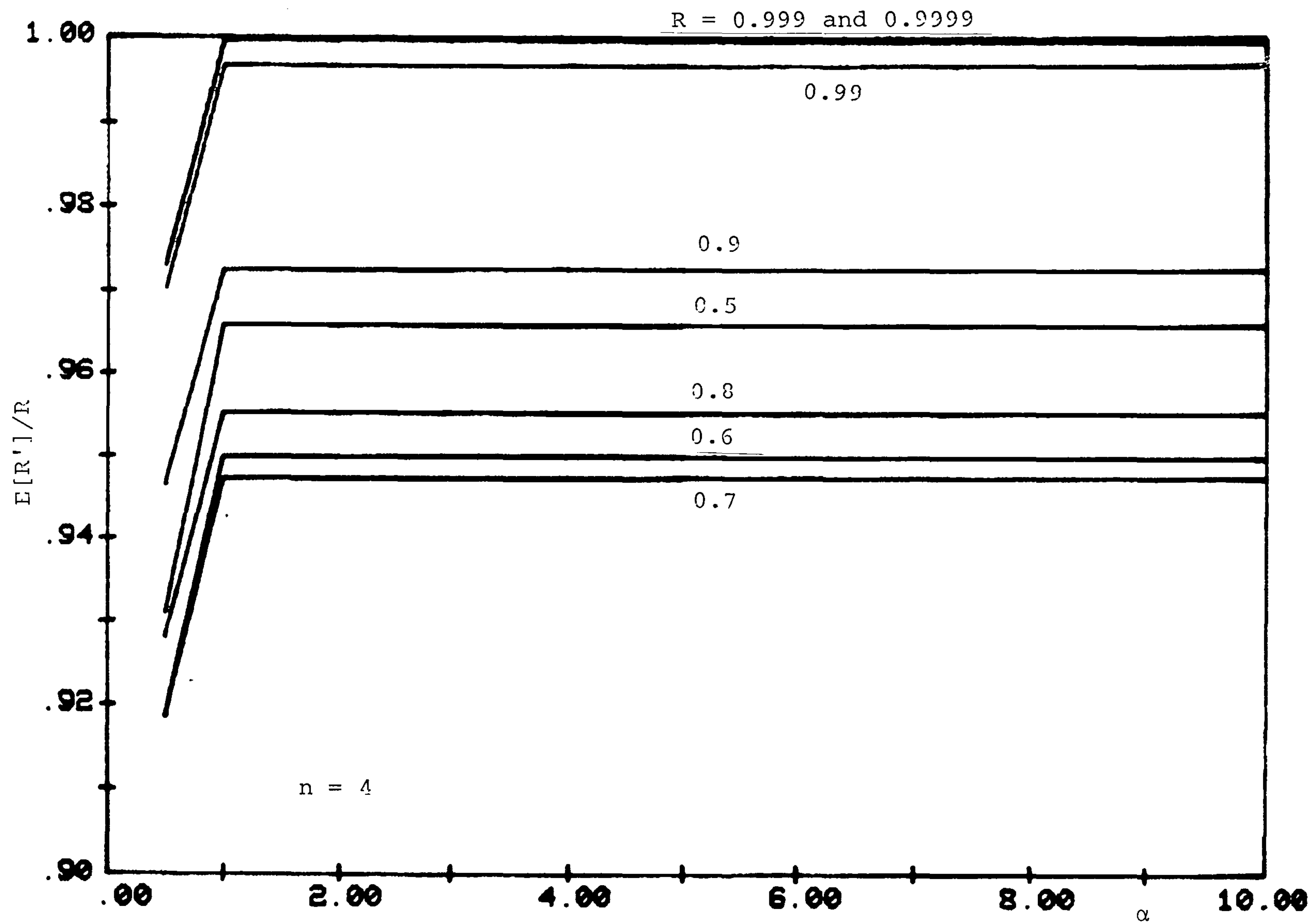

Fig. 6. E[R']/R as a function of α (n = 4)

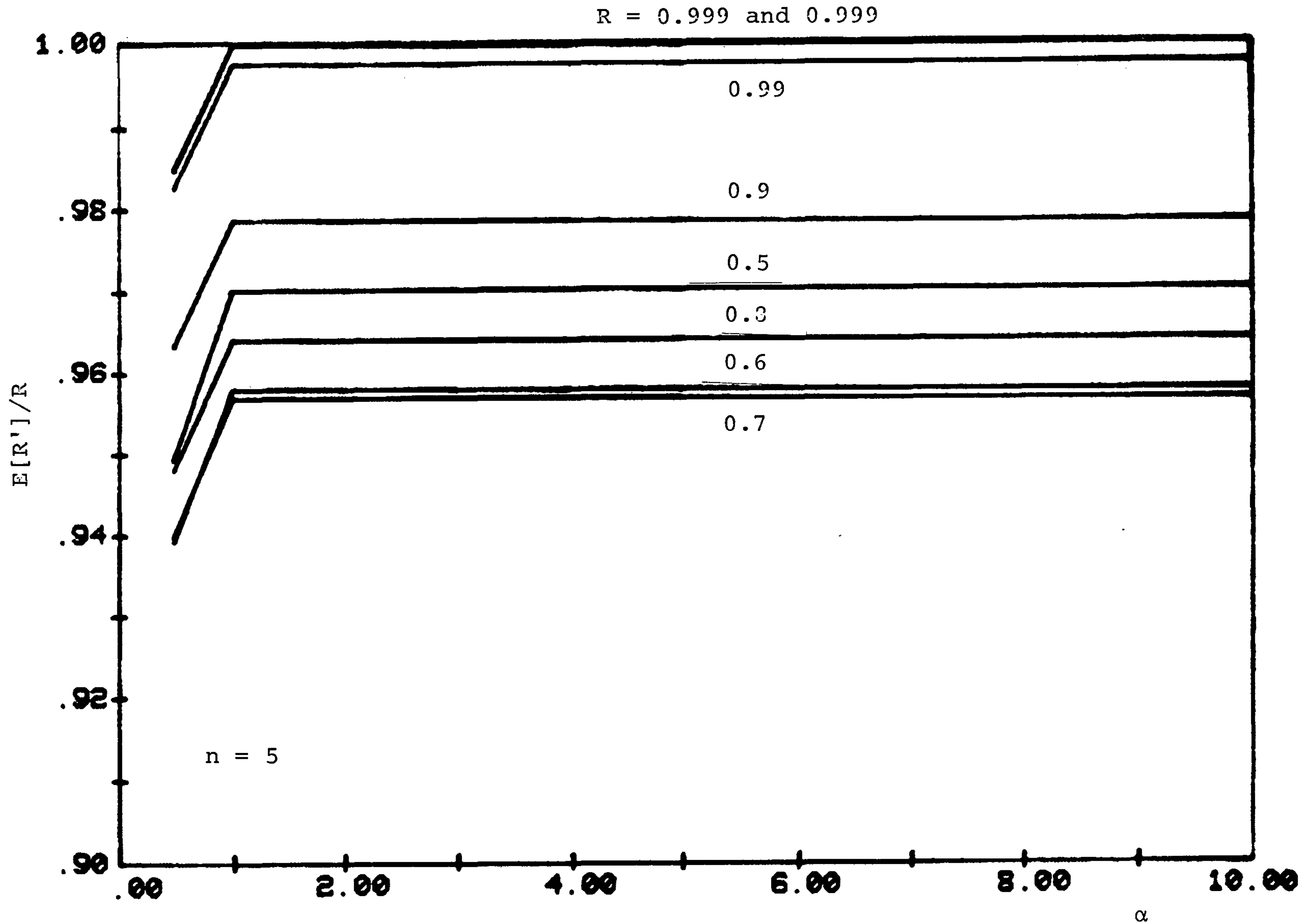

Fig. 7. E[R']/R as a function of α (n = 5)

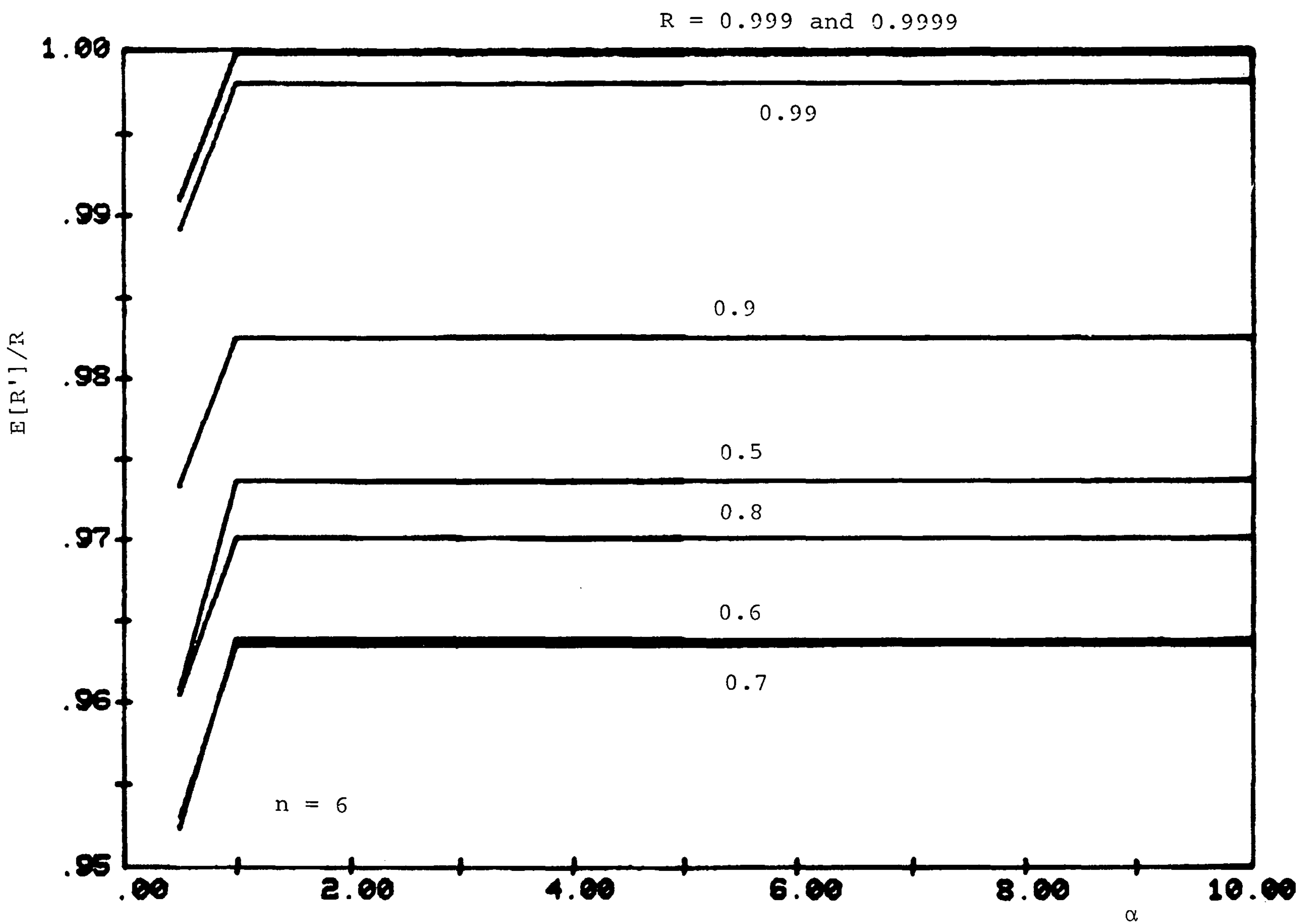

Fig. 8. E[R']/R as a function of α (n = 6)

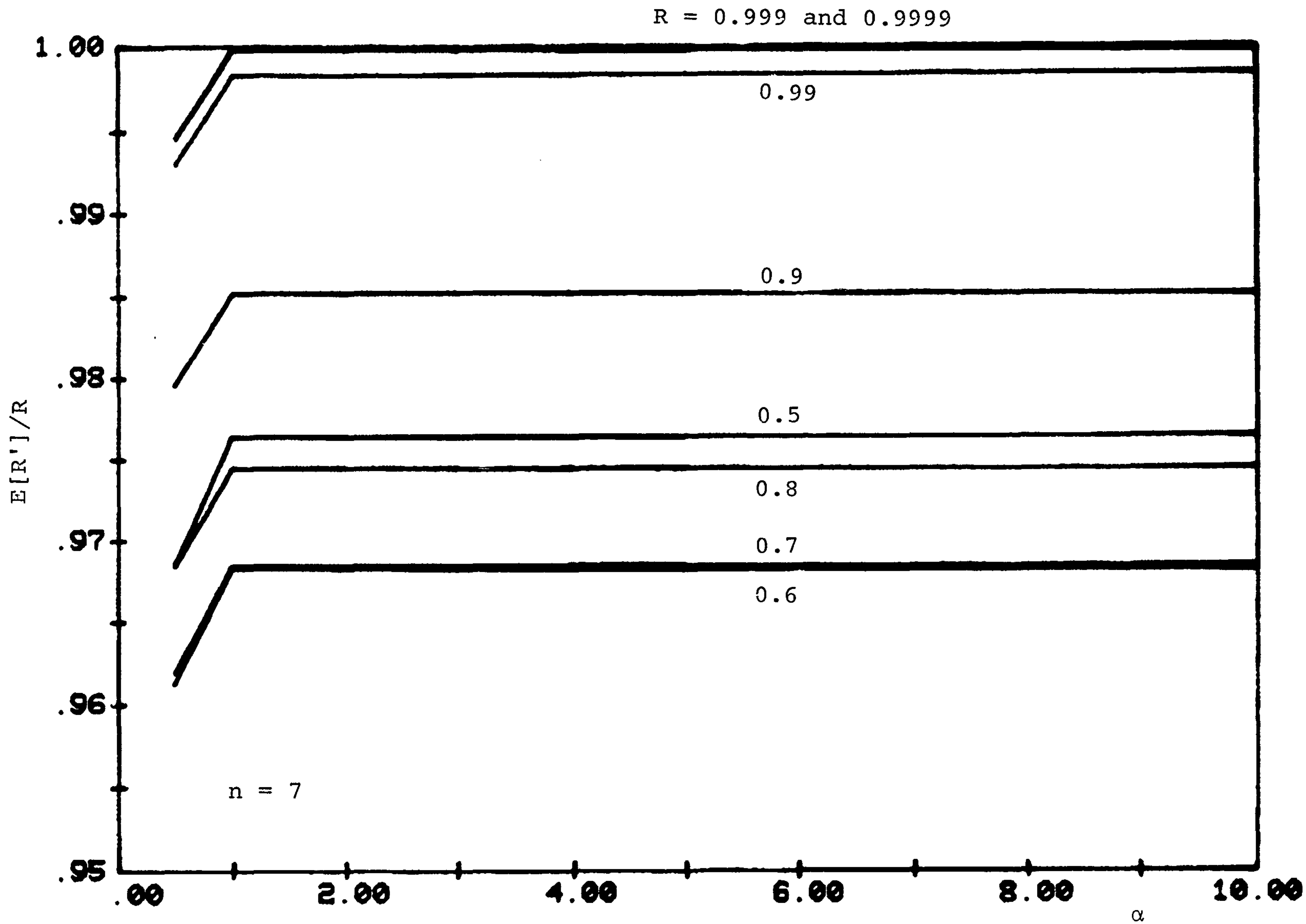

Fig. 9. E[R']/R as a function of α (n = 7)

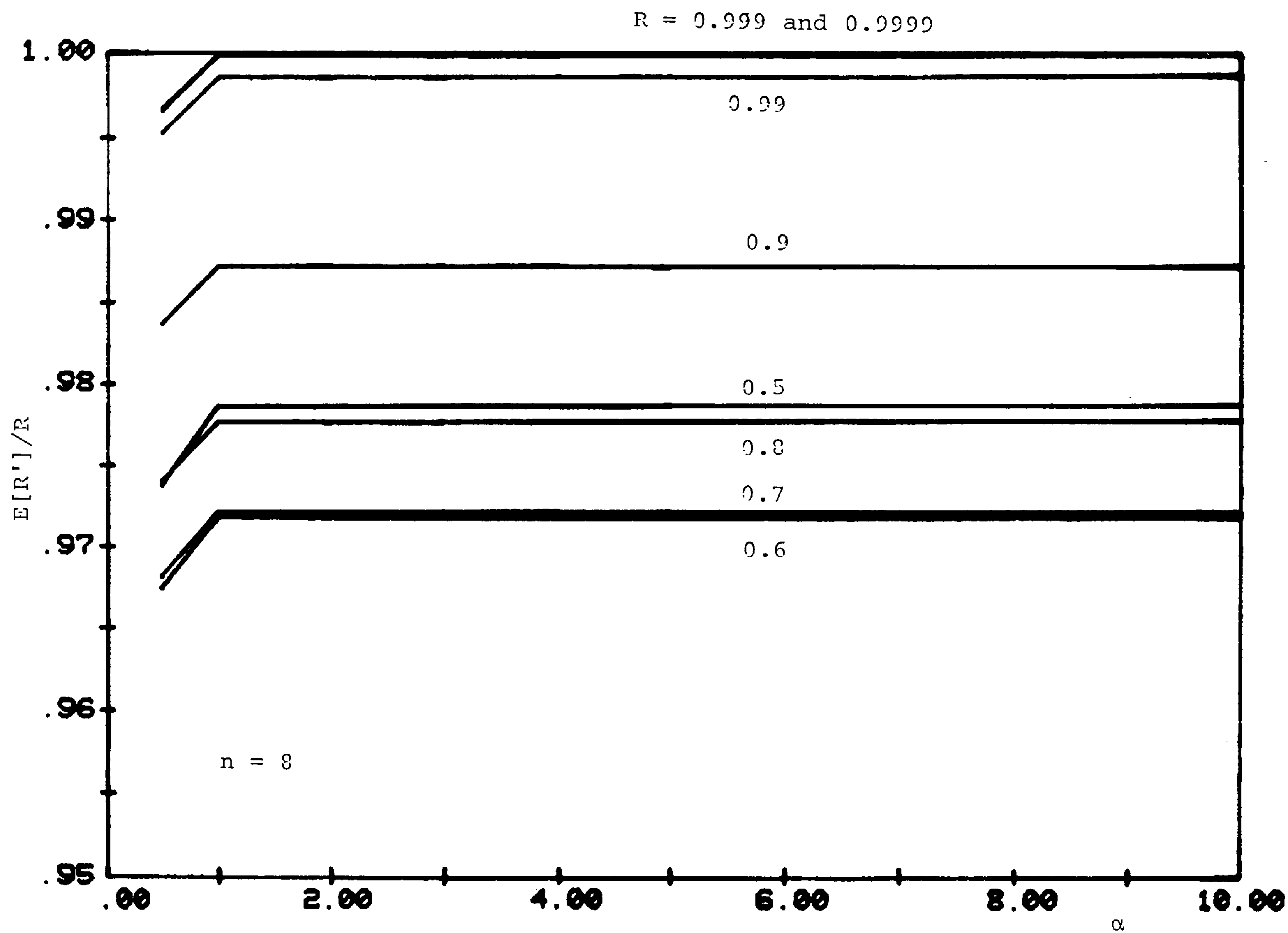

Fig. 10. E[R']/R as a function of α (n = 8)

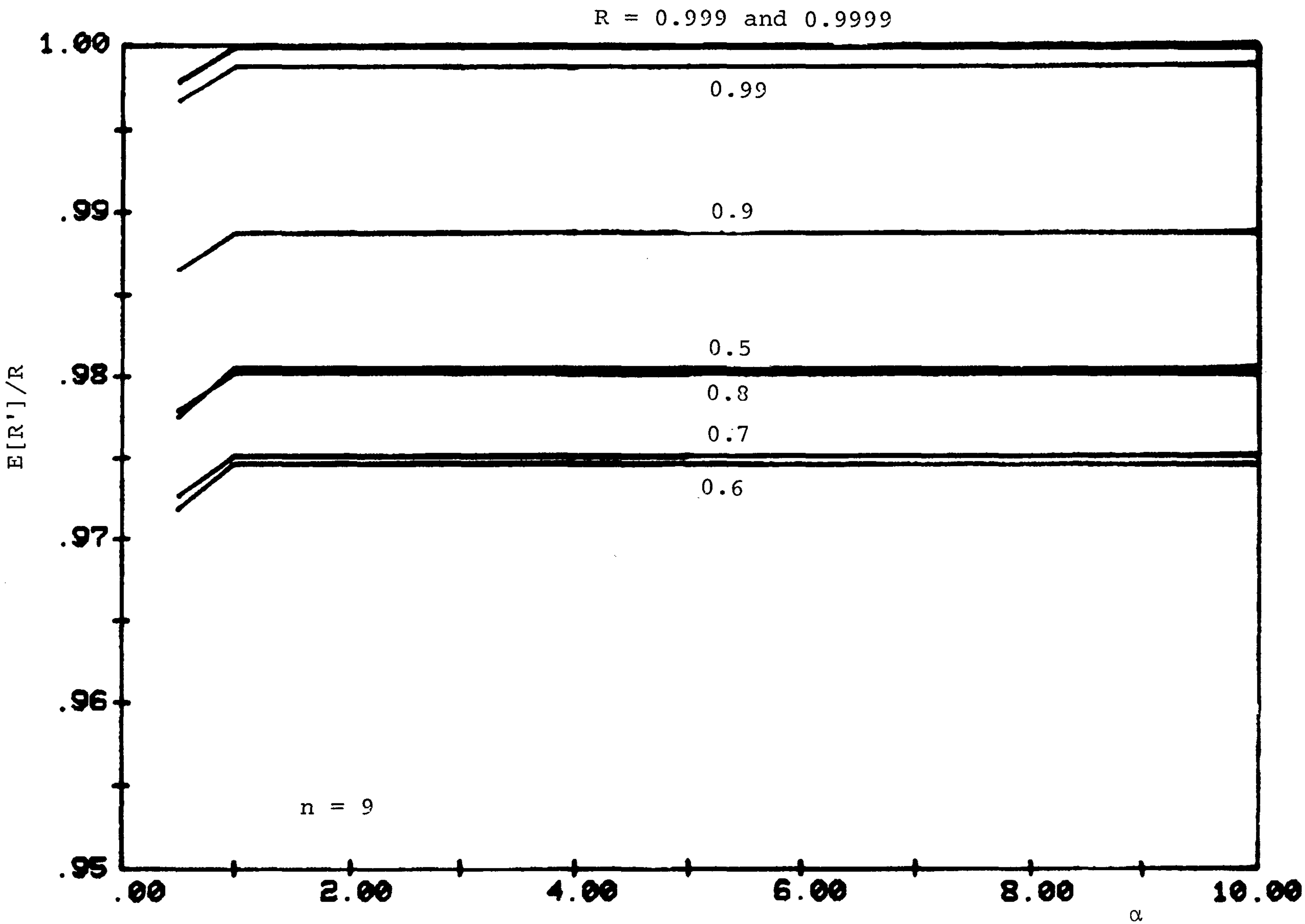

Fig. 11. E[R']/R as a function of α (n = 9)

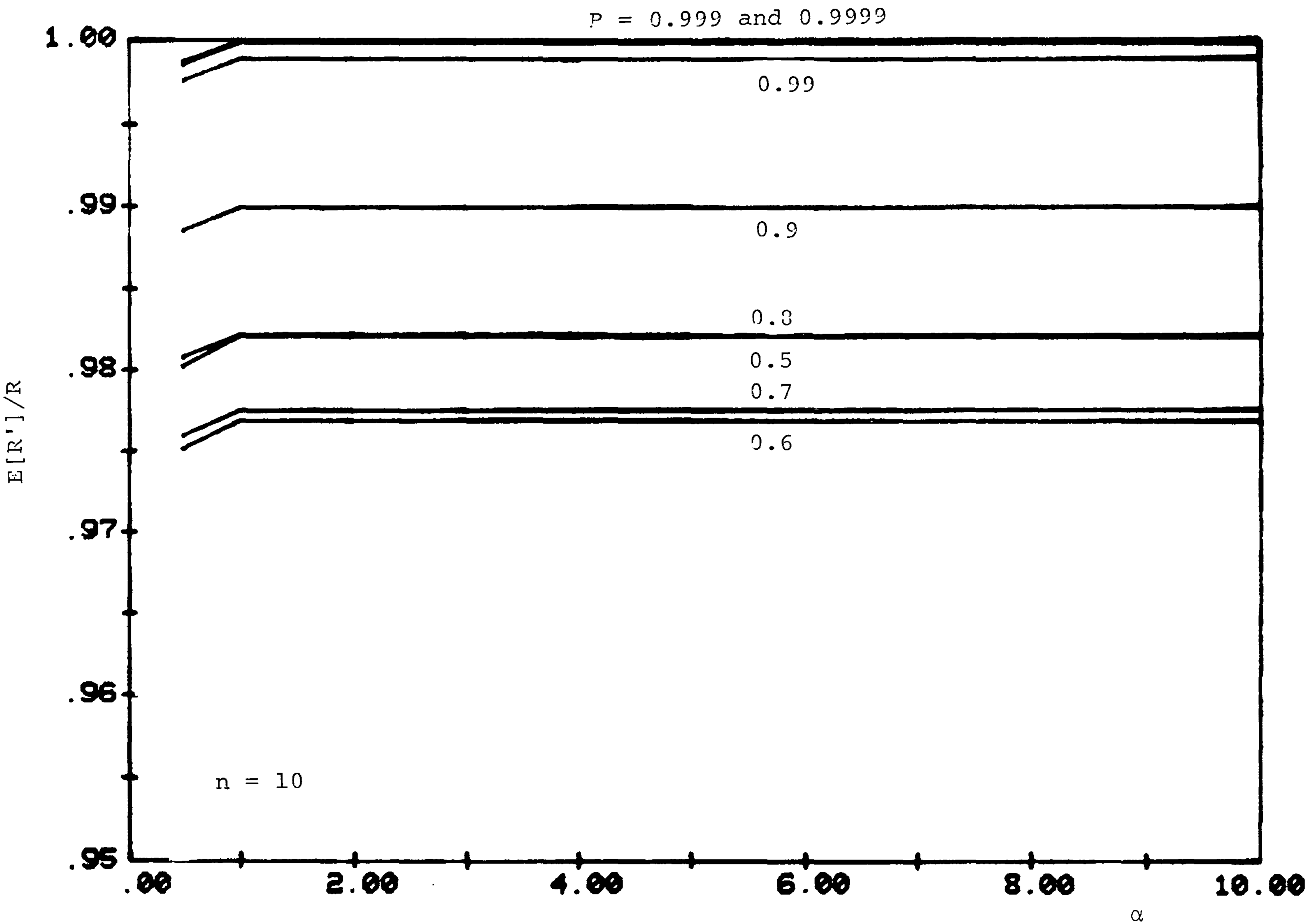

Fig. 12. E[R']/R as a function of α (n = 10)

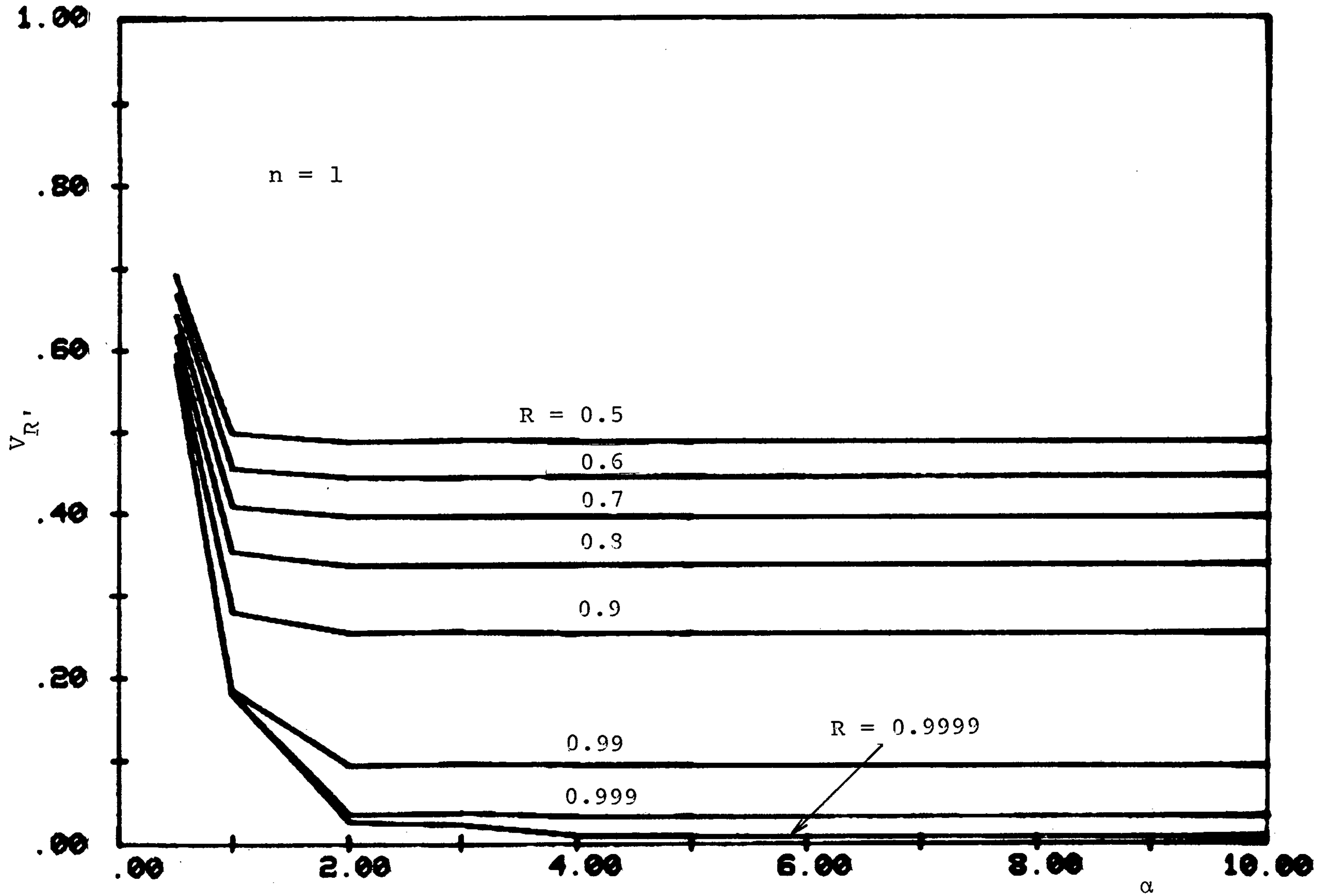

Fig. 13. $V_{R'}$ as a function of α (n = 1)

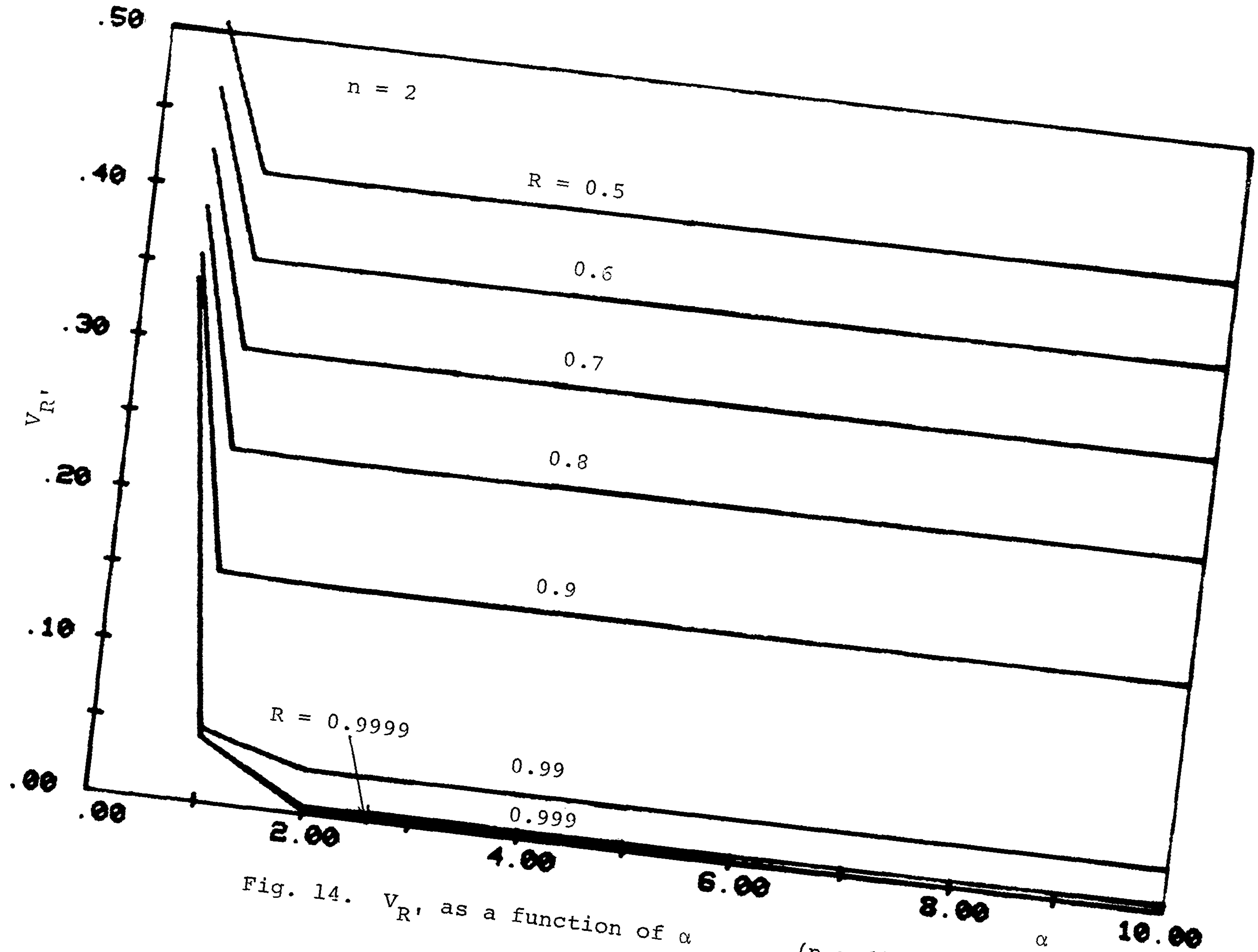

Fig. 14. $V_{R'}$ as a function of α (n = 2)

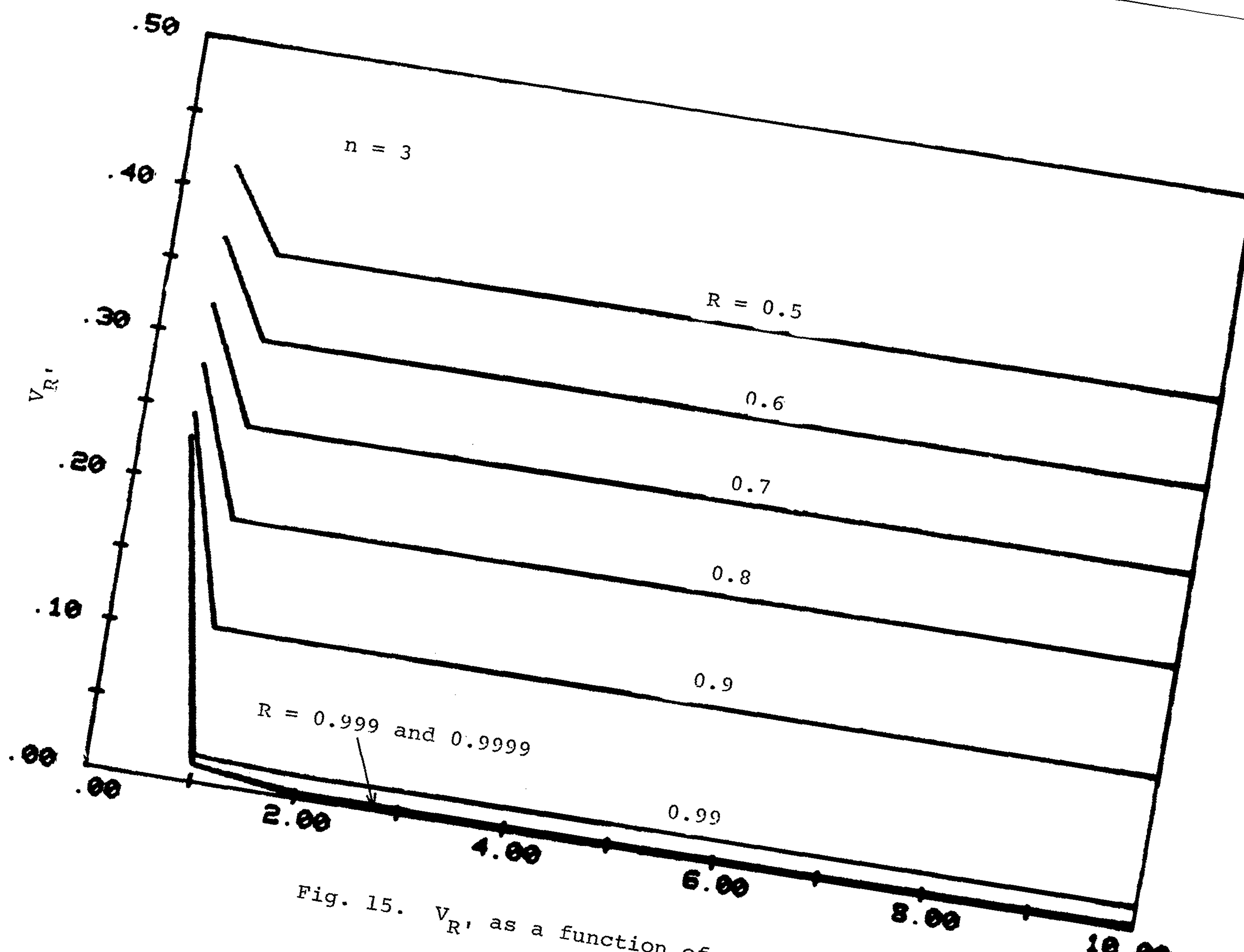

Fig. 15. $V_{R'}$ as a function of α (n = 3)

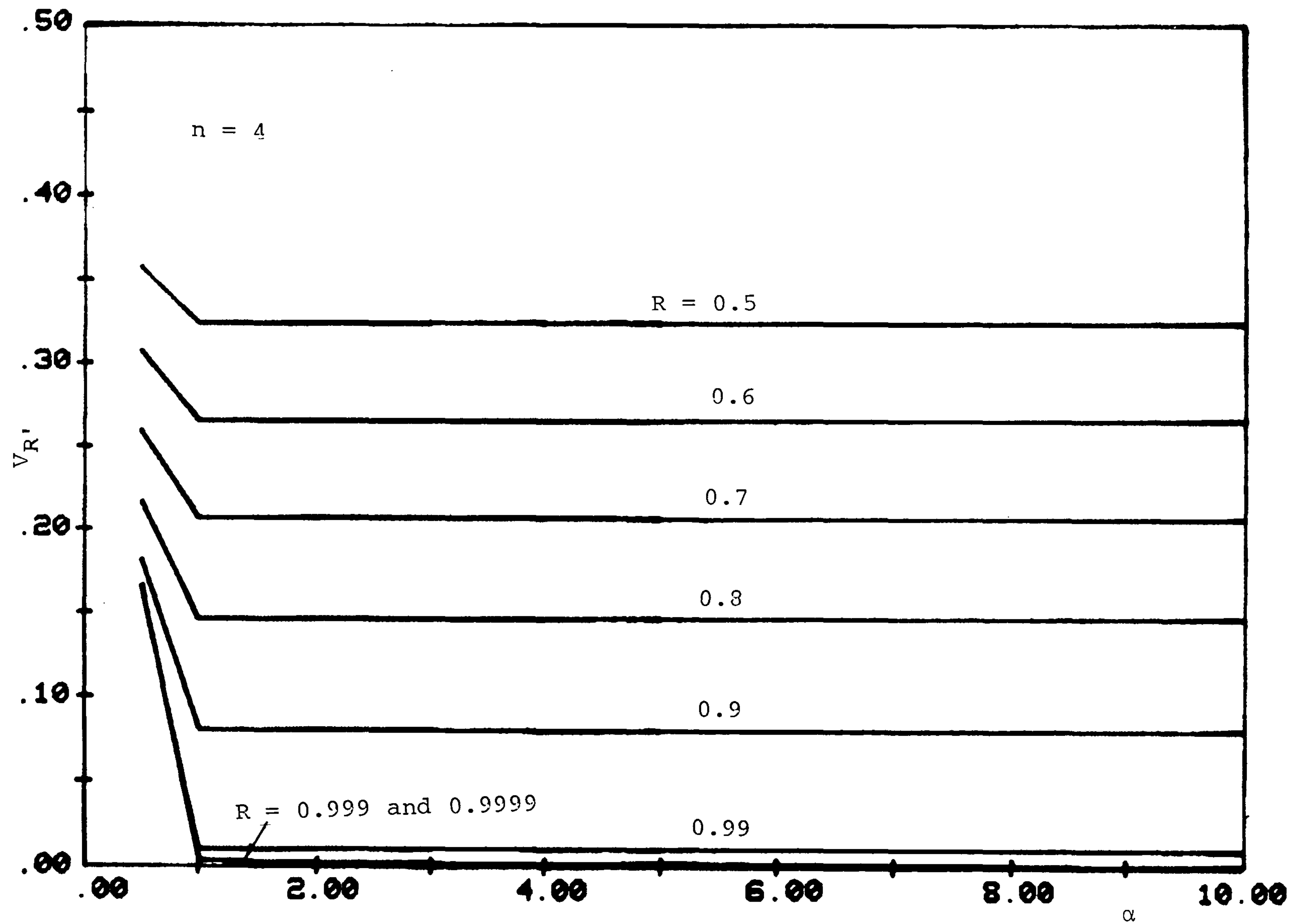

Fig. 16. $V_{R'}$ as a function of α (n = 4)

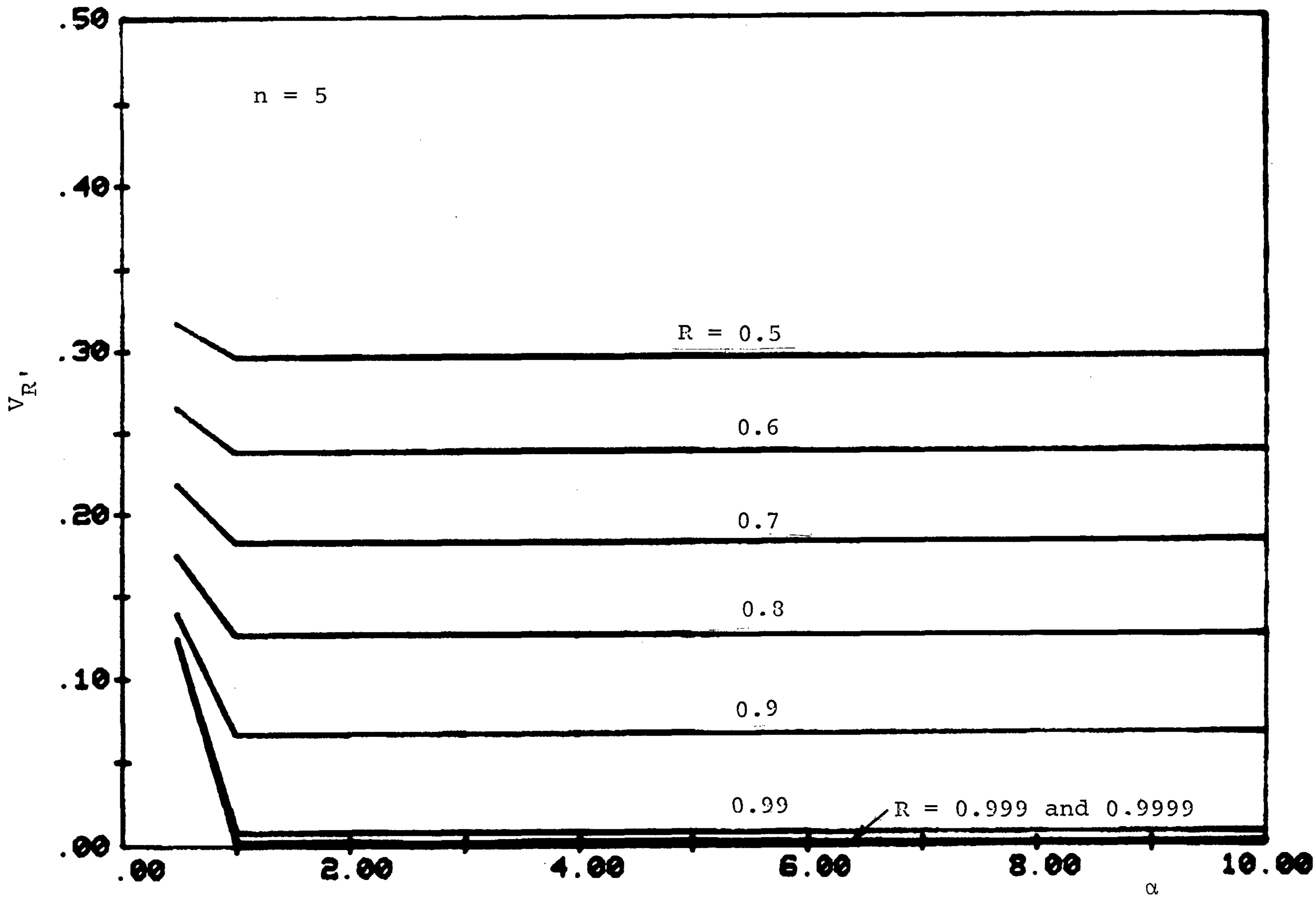

Fig. 17. $V_{R'}$ as a function of α (n = 5)

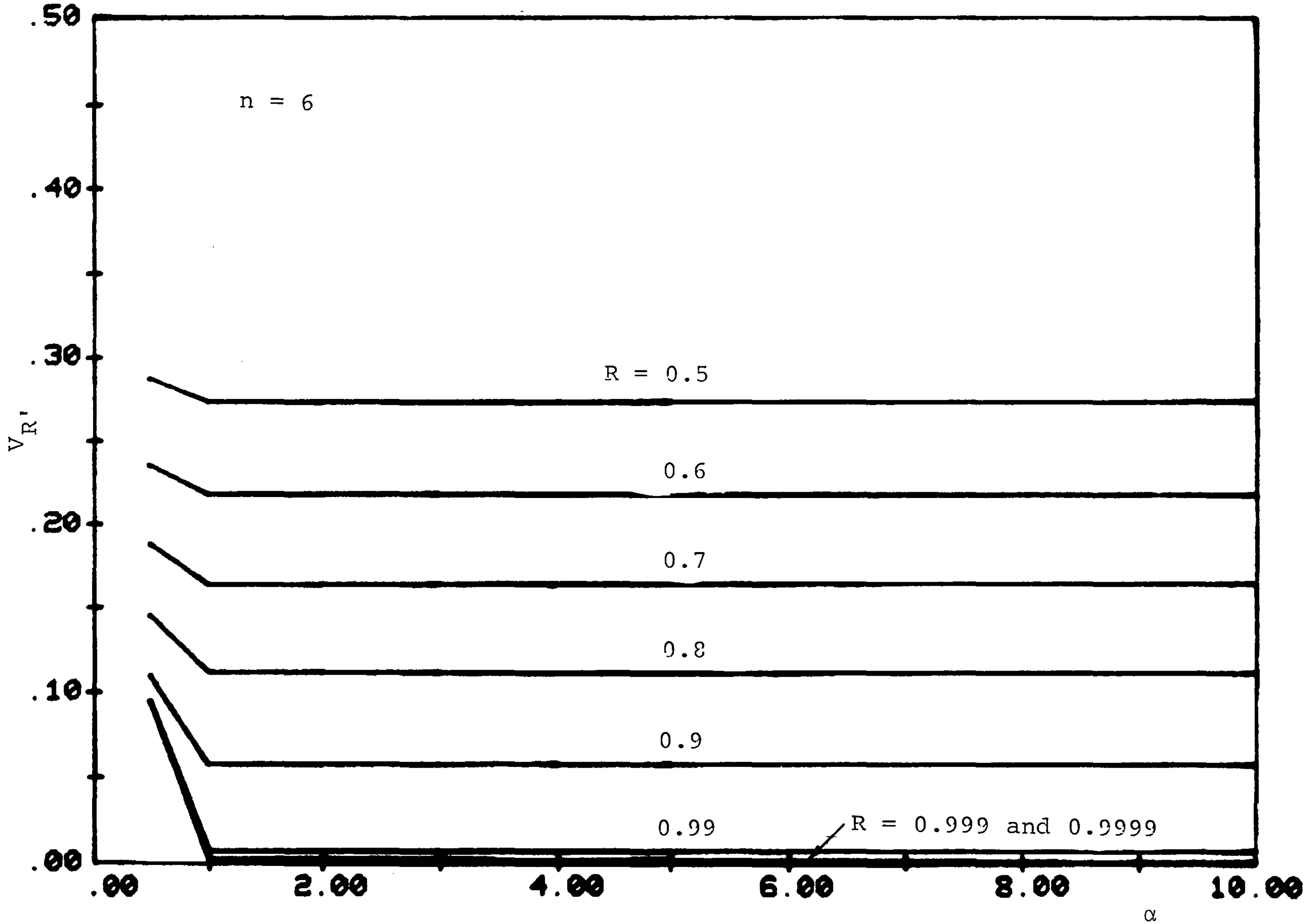

Fig. 18. $V_{R'}$ as a function of α (n = 6)

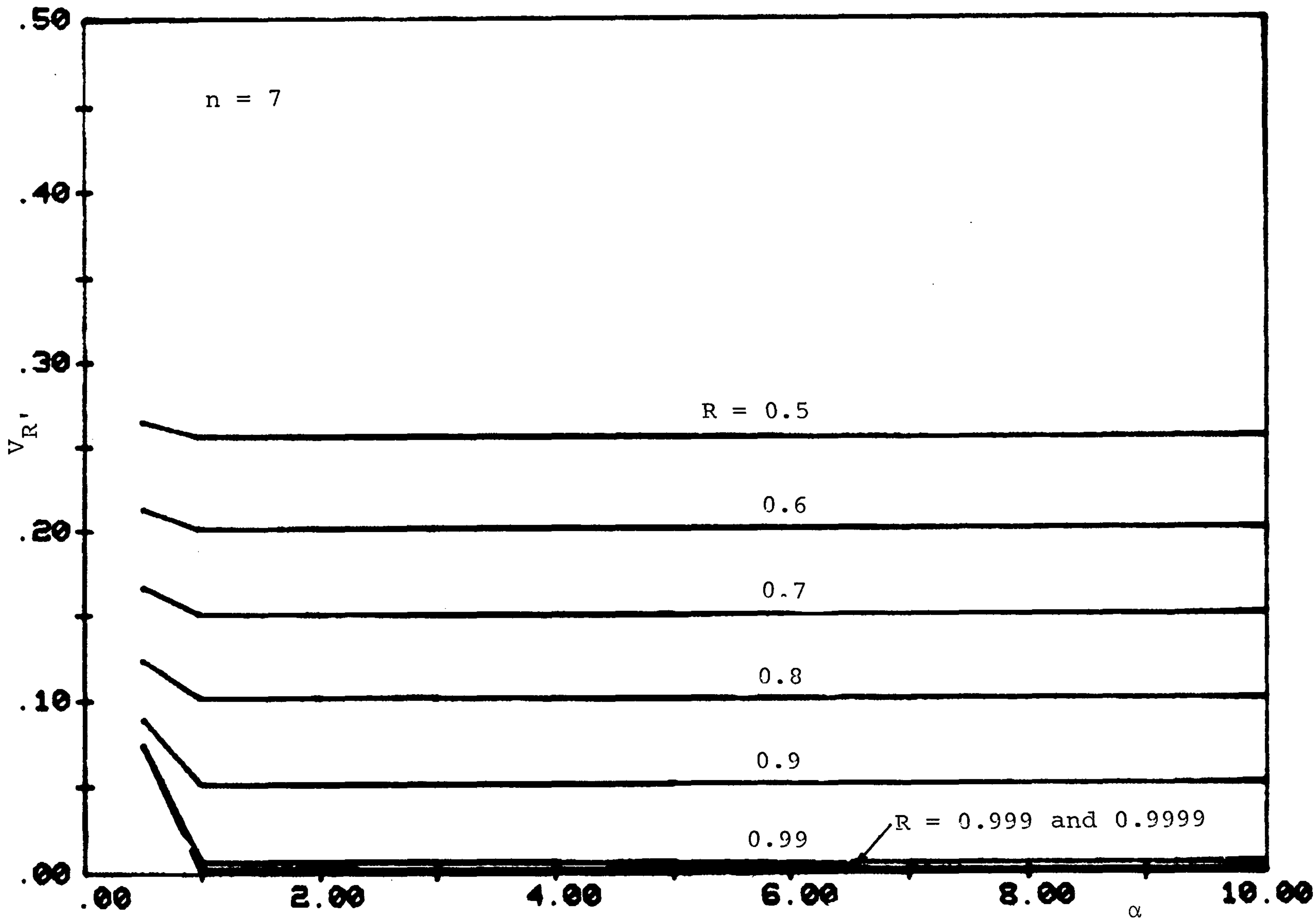

Fig. 19. $V_{R'}$ as a function of α (n = 7)

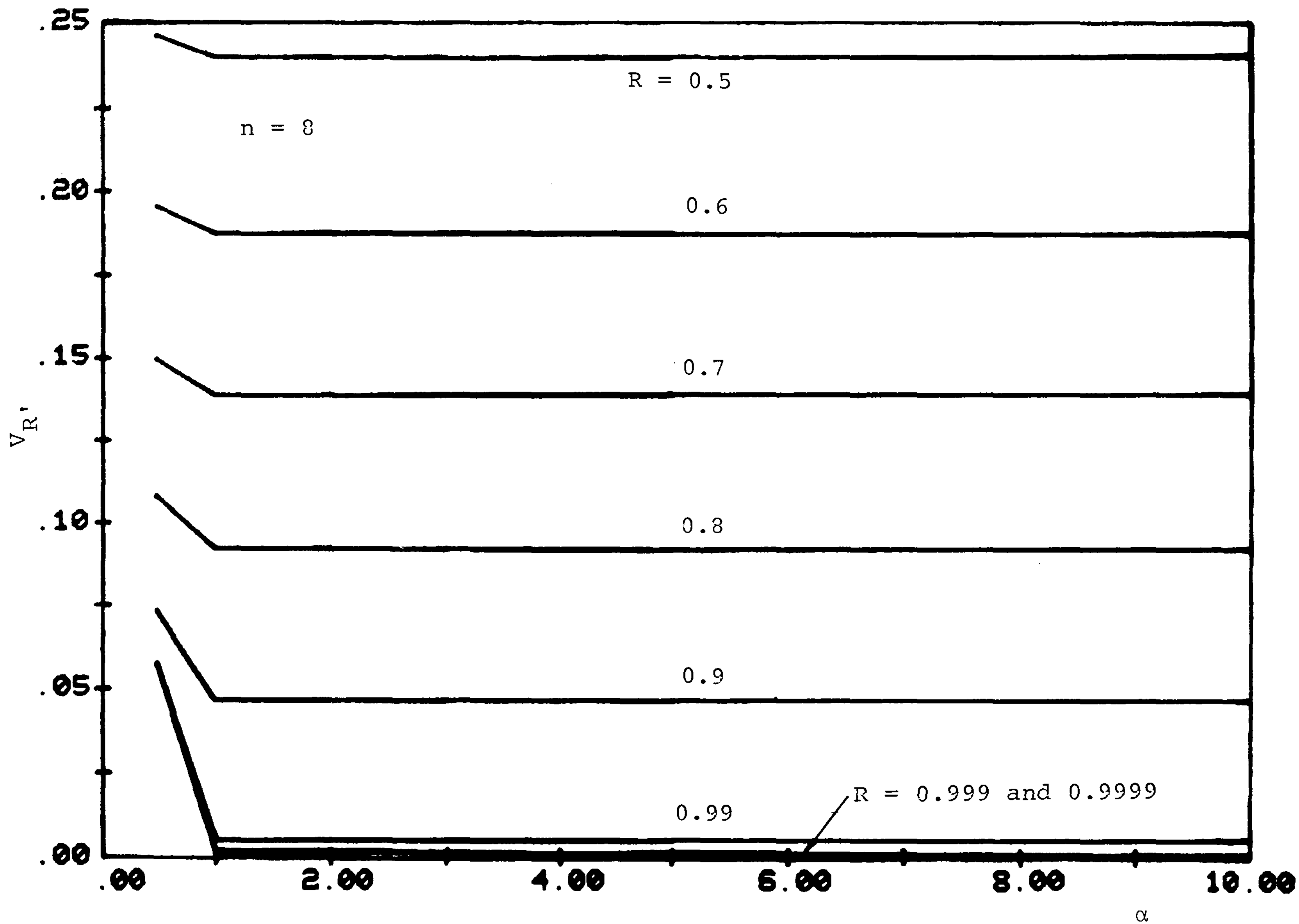

Fig. 20. V_R, as a function of α (n = 8)

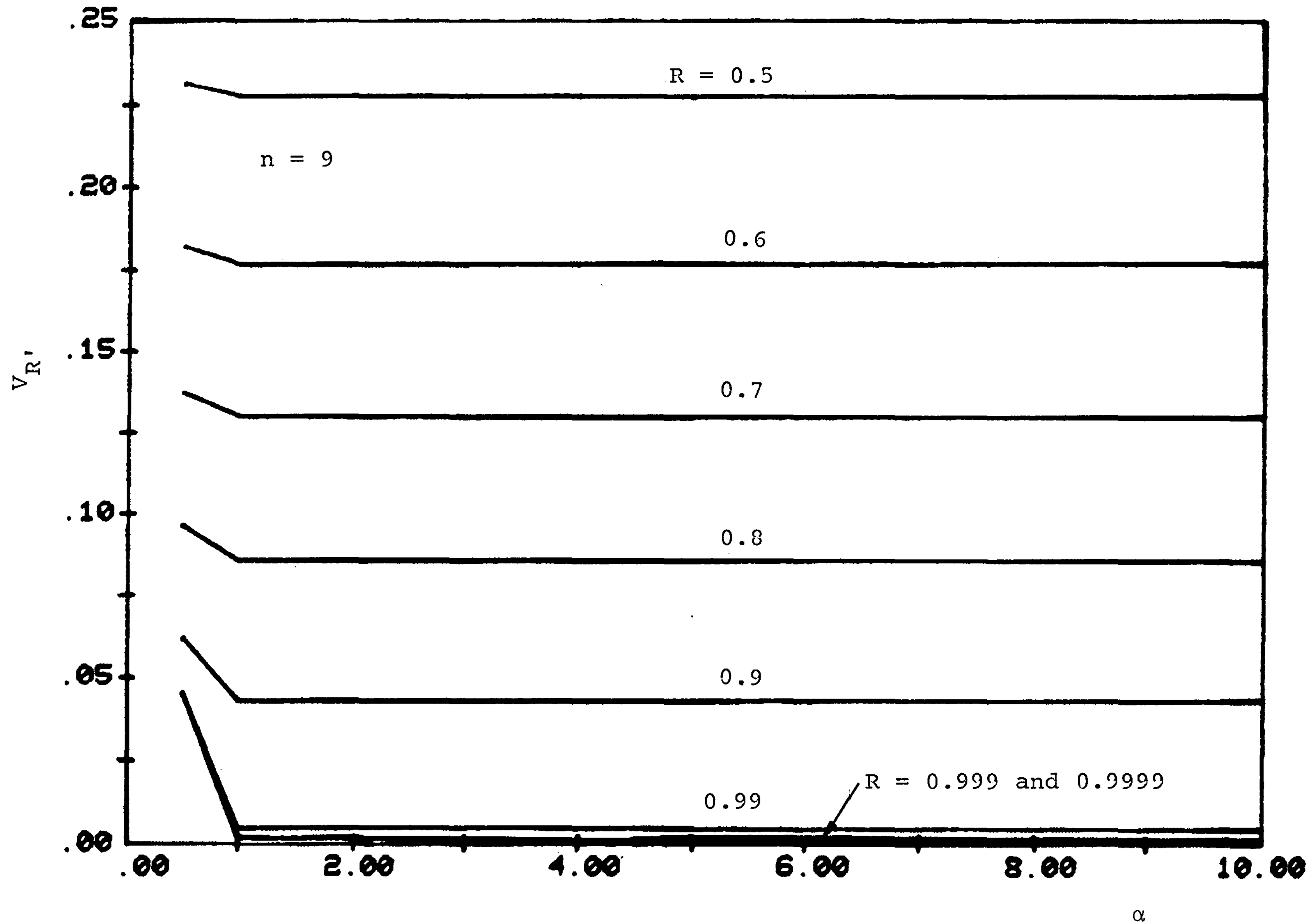

Fig. 21. $V_{R'}$ as a function of α (n = 9)

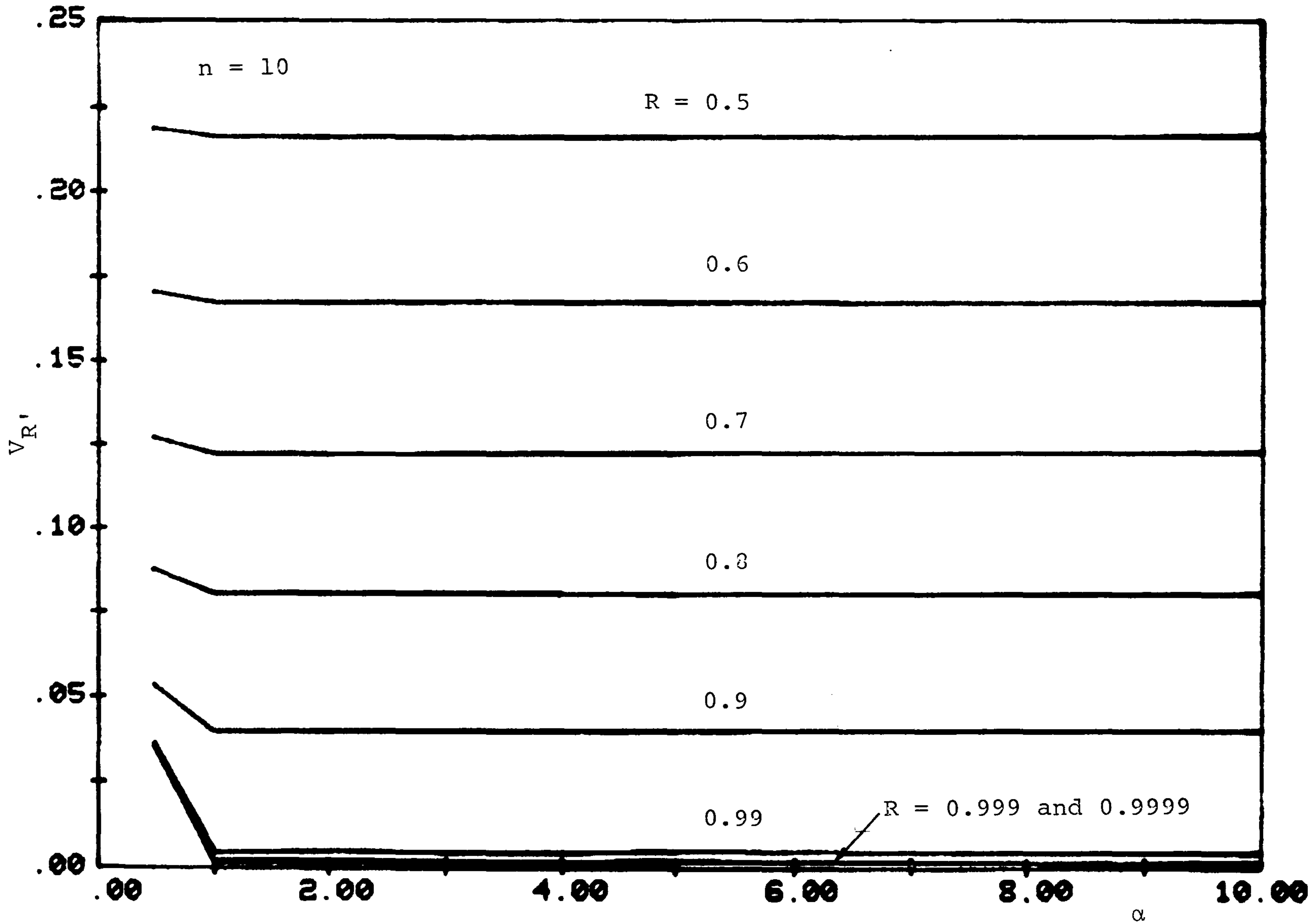

Fig. 22. $V_{R'}$ as a function of α (n = 10)

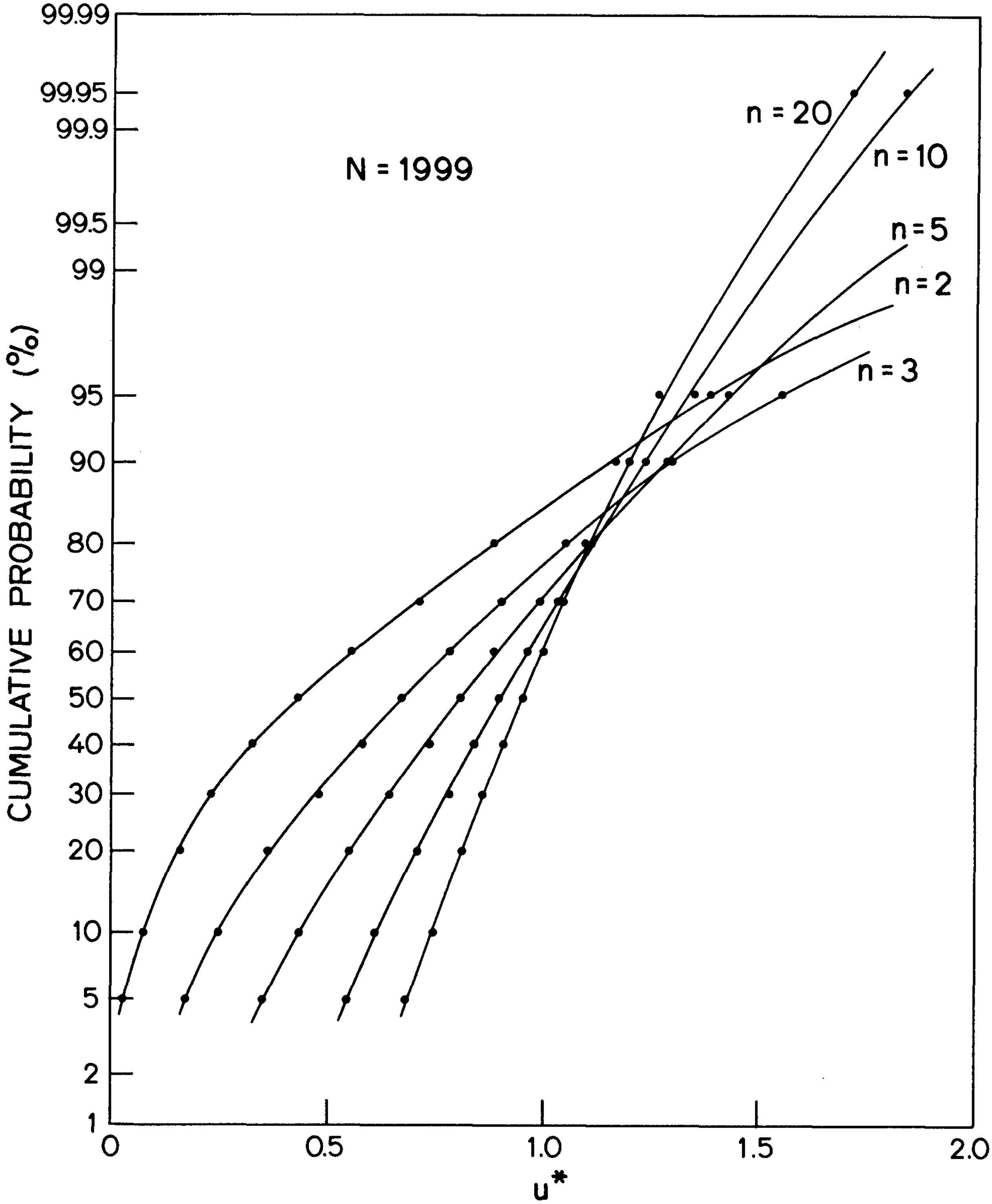

Fig. 23. Empirical distribution of u*.

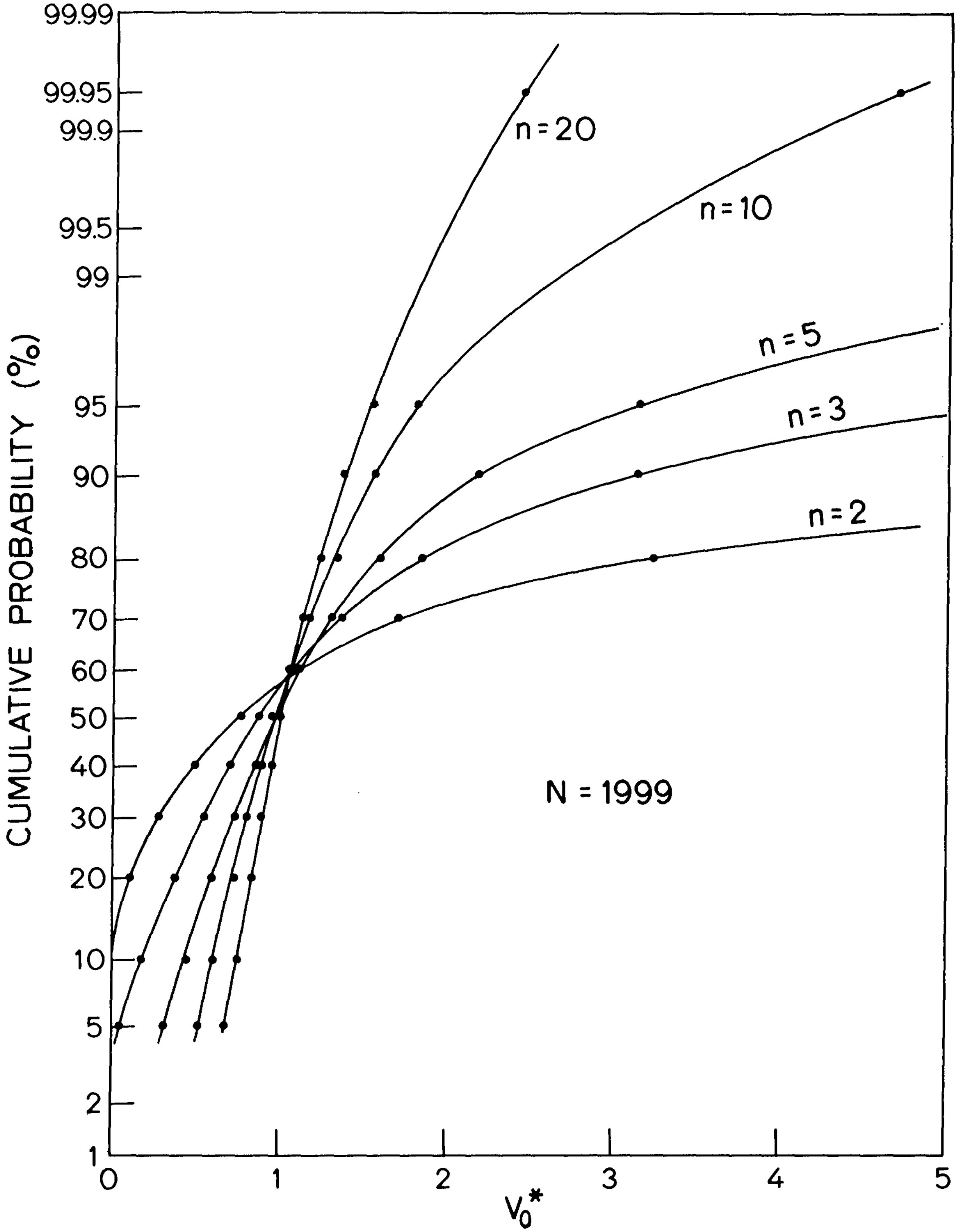

Fig. 24. Empirical distribution of V_0^*

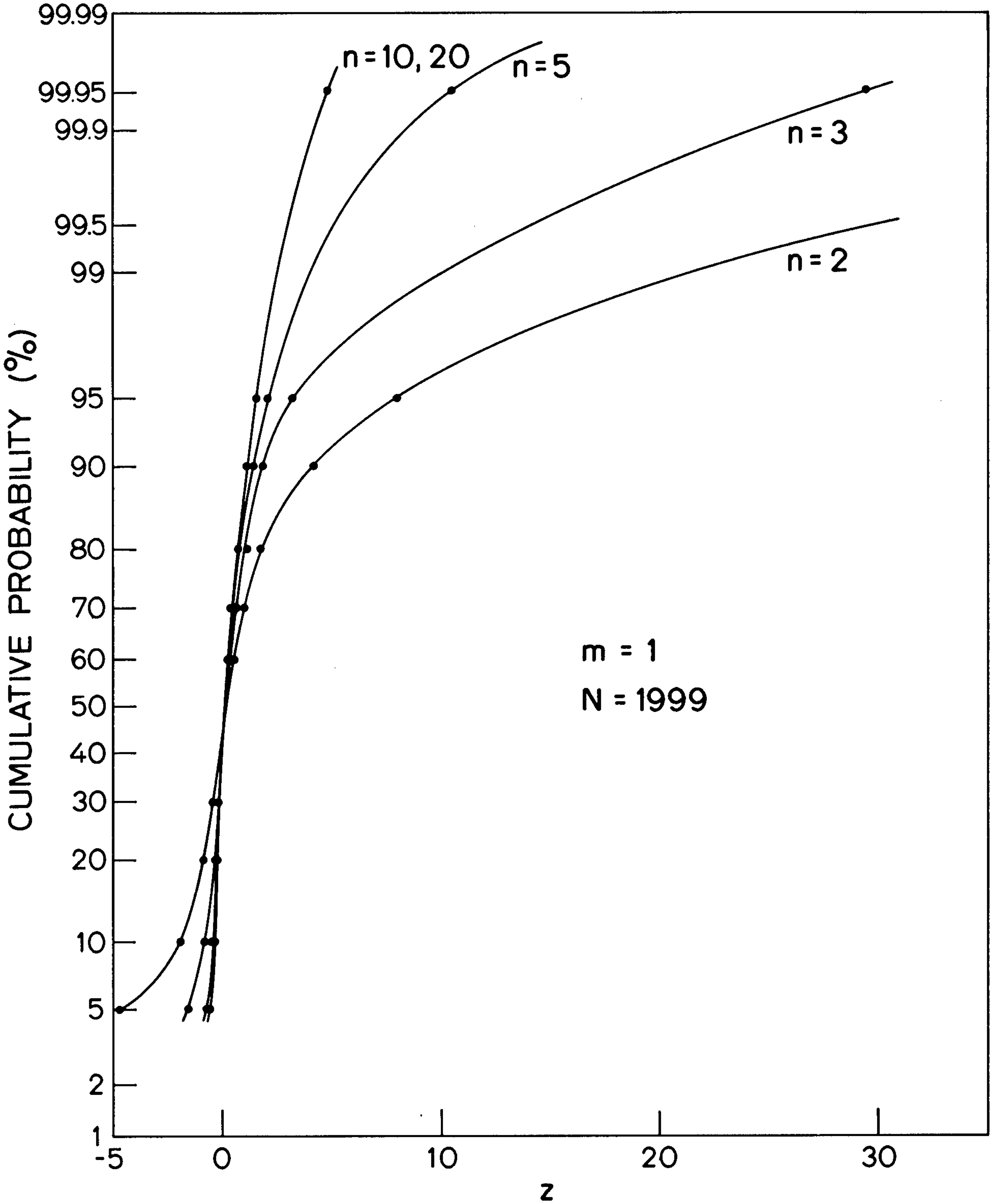

Fig. 25. Empirical distribution of Z (m = 1)

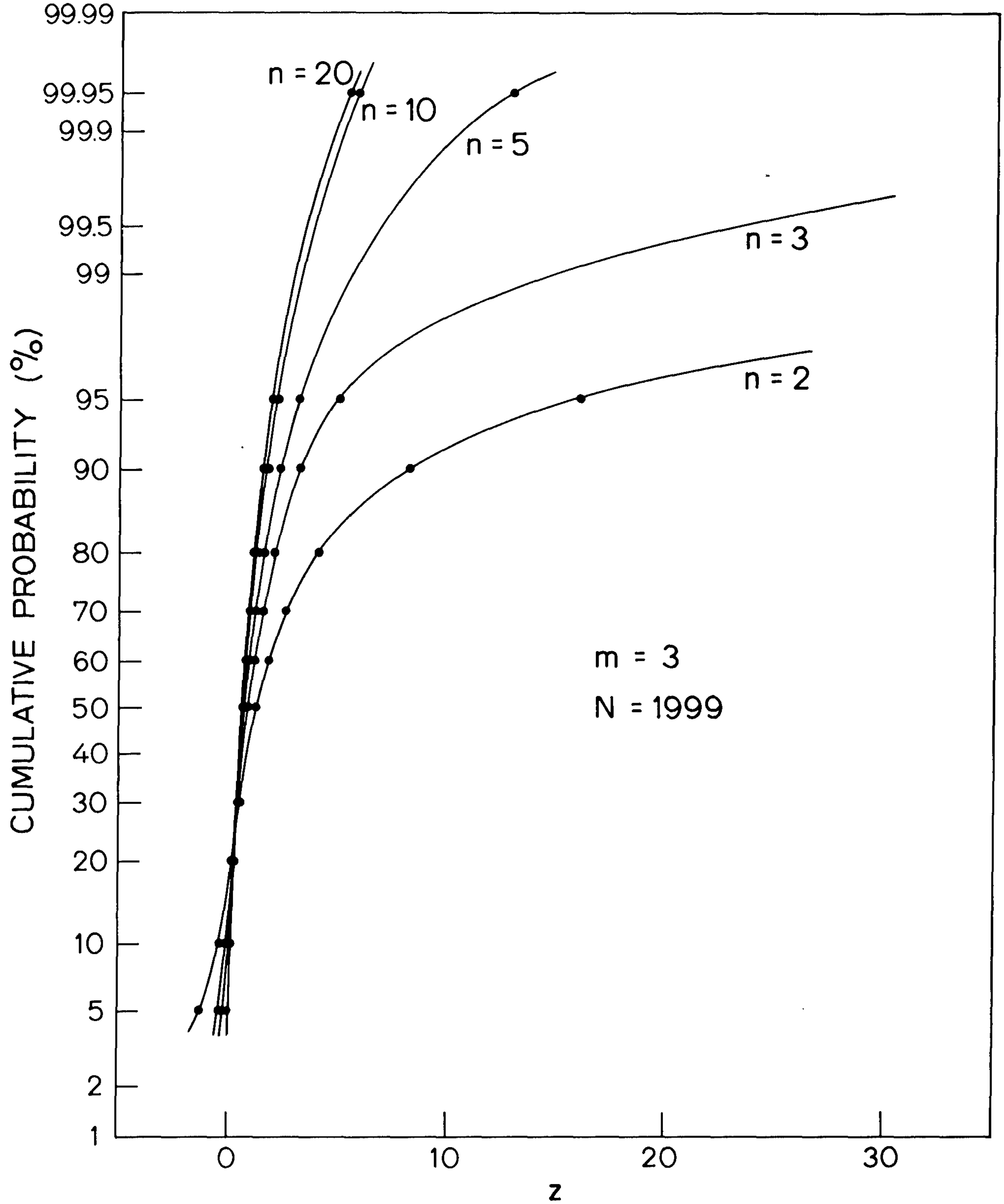

Fig. 26. Empirical distribution of Z (m=3)

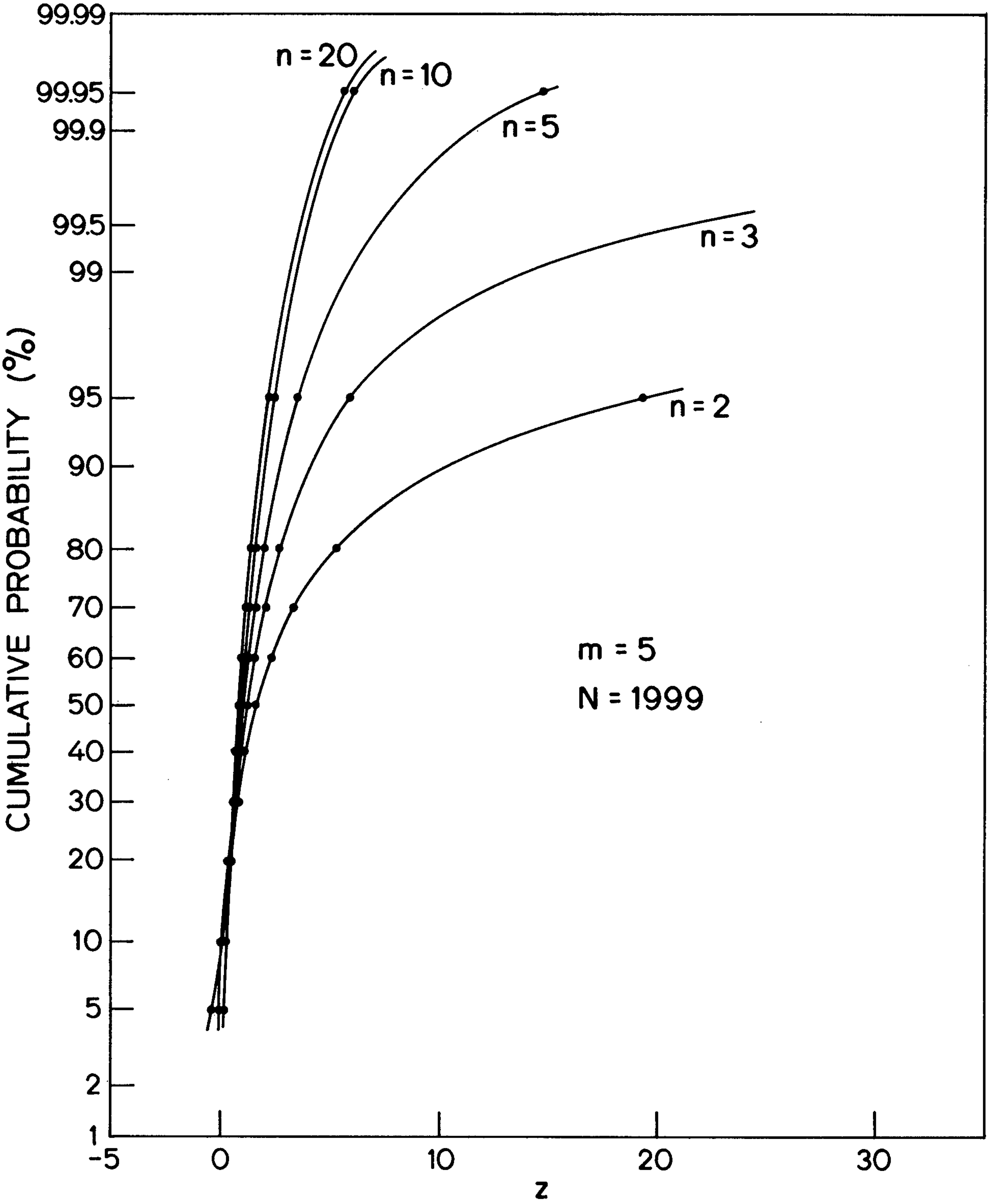

Fig. 27. Empirical distribution of Z (m = 5)

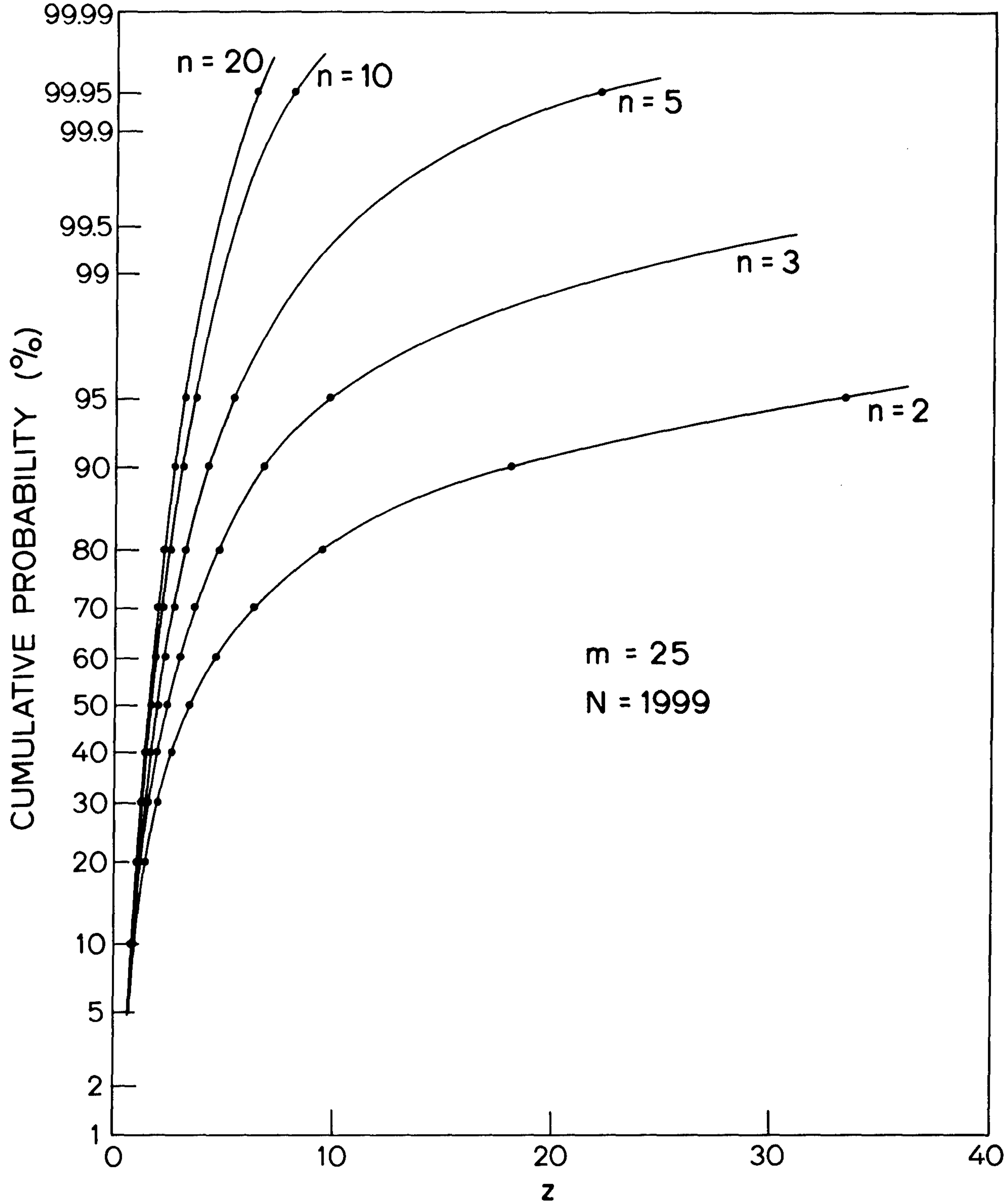

Fig. 28. Empirical distribution of Z (m = 25)

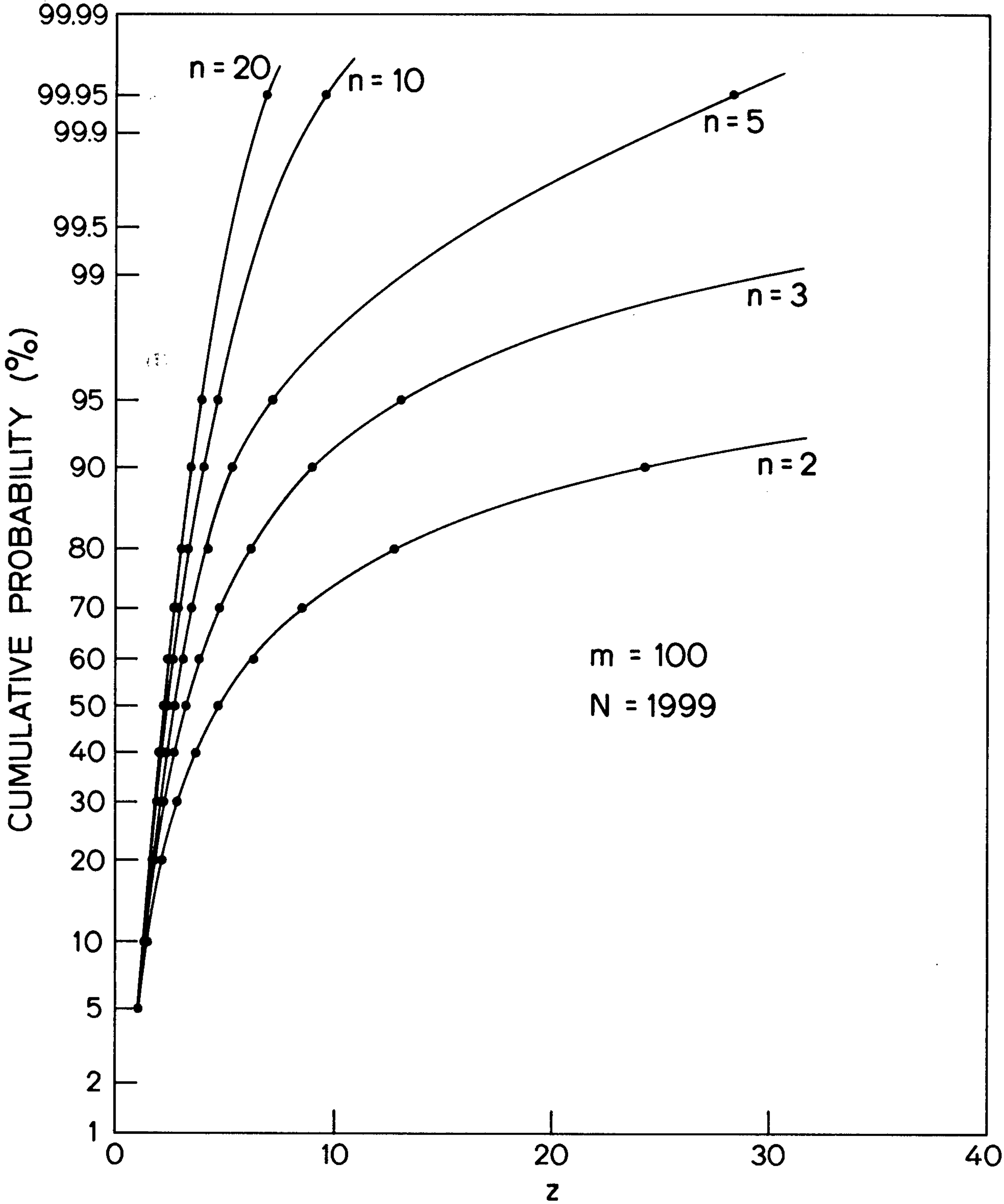

Fig. 29. Empirical distribution of Z (m=100)

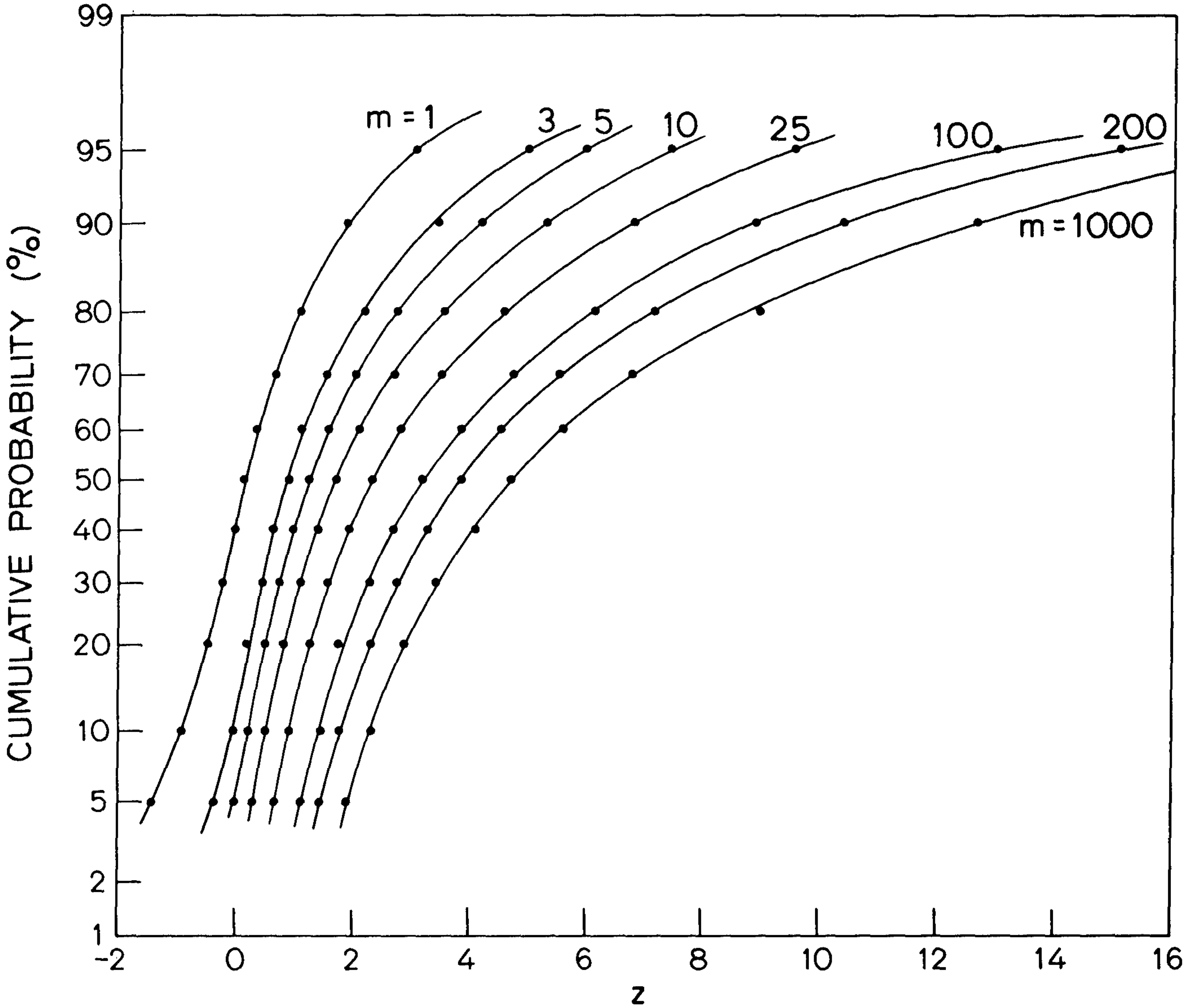

Fig. 30. Empirical distribution of Z (n = 3)

u \ v_0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	2	13	3	2	0	0	1	0	0	0	0	0	0	0	0	0
2	42	66	59	40	32	16	21	8	5	1	4	3	0	0	0	0
3	31	55	61	43	41	36	24	13	9	11	4	4	2	2	0	1
4	15	31	48	27	24	25	16	15	5	7	4	5	0	1	2	2
5	19	25	30	20	16	14	14	16	10	5	1	2	0	1	1	1
6	6	14	18	13	14	14	10	8	5	5	1	4	1	2	1	2
7	9	17	24	19	9	6	7	8	4	3	2	0	1	1	1	0
8	7	16	11	14	14	6	2	4	2	3	2	1	0	0	0	0
9	0	10	7	12	8	10	5	4	1	3	1	0	0	0	0	0
10	5	7	13	7	6	3	5	9	4	2	0	2	0	0	0	0
11	2	8	15	3	6	7	6	2	1	0	0	2	0	0	0	0
12	3	6	4	4	7	6	2	2	3	0	1	0	0	0	0	0
13	2	7	6	5	4	2	0	1	1	1	0	0	1	0	0	0
14	1	4	7	6	3	2	1	2	4	0	3	0	0	1	0	0
15	2	6	2	5	1	2	2	1	0	0	1	1	0	0	0	0
16	2	5	1	4	1	3	1	1	1	1	0	0	0	0	0	0
17	2	1	3	2	1	1	1	0	1	0	0	0	2	0	0	0
18	2	3	3	3	1	1	0	0	0	0	0	0	0	0	0	0
19	1	2	4	2	1	1	1	1	2	0	1	1	0	0	0	0
20	0	0	1	1	1	3	0	0	0	1	0	0	0	0	0	0
21	1	2	3	2	3	3	0	1	0	0	0	0	0	0	0	0
22	0	2	0	0	0	2	2	1	0	0	0	1	0	0	0	0
23	2	1	2	2	2	0	0	0	1	0	0	0	0	0	0	0
24	0	0	1	1	2	0	0	0	0	0	0	0	0	0	0	0
25	1	2	2	5	3	1	4	1	0	0	0	0	0	0	0	0
26	1	2	0	0	0	2	1	0	0	0	1	0	0	0	0	0
27	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0
28	0	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0
29	1	0	2	2	2	2	0	0	0	0	0	0	0	0	0	0
30	2	0	0	2	0	0	1	0	1	0	0	0	0	0	0	0
31	1	1	0	0	0	0	0	0	2	0	0	0	0	0	0	0
32	1	1	2	0	1	0	0	2	0	0	0	0	0	0	0	0
33	0	1	1	1	0	2	0	1	0	0	0	0	0	0	0	0
34	0	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0
35	0	1	0	0	1	0	0	0	0	0	0	0	0	1	0	0
36	0	1	0	0	1	0	0	0	0	0	0	1	0	0	0	0
37	0	3	1	0	0	0	0	0	0	0	0	1	0	0	0	0
38	1	1	2	2	0	0	0	0	0	0	0	0	0	0	0	0
39	0	1	1	0	2	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Notes: For v_0, Interval 1 = 0 - 0.25, Interval 2 = 0.25 - 0.50, etc.
For u, Interval 1 = 0 - 0.50, Interval 2 = 0.50 - 1.00, etc.

Fig. 31. Two-dimensional frequency of u and v_0 (n=2)

u \ v_0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	6	3	1	1	1	0	0	0	0	0	0	0	0	0	0
2	30	104	89	82	59	48	19	16	5	4	2	0	2	1	0	0
3	22	64	96	102	90	61	44	17	17	14	4	0	2	2	1	0
4	11	46	68	54	57	33	33	21	10	11	6	3	1	0	1	0
5	5	25	21	29	34	18	14	12	3	6	3	0	1	0	0	0
6	5	11	20	9	22	12	11	8	4	3	1	1	1	0	1	1
7	3	13	27	14	12	4	13	5	4	1	0	0	0	0	0	0
8	3	8	11	12	5	8	3	3	1	3	1	0	1	0	0	0
9	1	4	10	10	6	3	3	2	2	0	0	1	0	0	0	0
10	2	11	3	4	8	1	1	0	1	0	0	0	0	0	0	0
11	1	1	1	3	3	3	1	1	0	2	0	0	0	0	0	0
12	0	1	2	5	3	2	2	1	0	0	0	0	0	0	0	0
13	1	2	3	3	1	2	1	0	1	0	1	0	0	0	0	0
14	0	1	2	2	3	1	1	0	0	0	0	0	0	1	0	0
15	0	0	2	2	1	0	1	0	0	0	0	0	0	0	0	0
16	0	0	1	2	1	0	0	1	0	0	0	0	0	0	0	0
17	0	1	1	2	1	0	0	0	0	0	0	0	0	0	0	0
18	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0
19	0	0	1	0	2	1	0	0	0	0	0	1	0	0	0	0
20	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0
22	1	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0
24	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
25	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
26	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0
28	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	0
29	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
33	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0

Notes: For v_0, Interval 1 = 0 - 0.25, Interval 2 = 0.25 - 0.50, etc.
For u, Interval 1 = 0 - 0.50, Interval 2 = 0.50 - 1.00, etc.

Fig. 32. Two-dimensional frequency of u and v_0 (n=3)

$u \backslash v_0$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	12	74	136	142	111	48	32	10	3	0	2	0	0	0	0	0
3	5	50	156	180	158	95	66	31	11	9	4	3	0	1	1	0
4	4	18	61	61	72	52	38	25	10	9	3	1	2	0	0	0
5	1	7	26	29	36	23	10	11	1	2	2	1	0	0	0	0
6	1	4	11	14	15	8	10	5	2	0	1	0	0	0	0	0
7	0	3	4	4	8	3	2	4	0	2	0	0	0	0	0	0
8	0	2	2	9	4	1	1	1	0	0	0	0	0	0	0	0
9	0	1	1	2	1	5	2	0	0	0	0	0	0	0	0	0
10	0	2	0	0	0	1	2	1	0	0	0	0	0	0	0	0
11	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	1	1	2	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	1	2	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Notes: for v_0, Interval 1 = 0 - 0.25, Interval 2 = 0.25 - 0.50, etc.
for u, Interval 1 = 0 - 0.50, Interval 2 = 0.50 - 1.00, etc.

Fig. 33. Two-dimensional frequency of u and v_0 (n=5)

u \ v_0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	35	186	238	148	68	13	1	2	1	0	0	0	0	0	0
3	2	28	146	318	260	161	68	22	6	3	0	1	0	0	0	0
4	0	3	30	69	57	45	28	11	1	0	0	0	0	0	0	0
5	0	0	4	11	6	9	2	1	1	1	0	0	0	0	0	0
6	0	0	0	4	2	3	0	1	0	0	0	0	0	0	0	0
7	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Notes: for v_0, Interval 1 = 0 - 0.25, Interval 2 = 0.25 - 0.50, etc.
for u , Interval 1 = 0 - 0.50, Interval 2 = 0.50 - 1.00, etc.

Fig. 34. Two-dimensional frequency of u and v_0 (n=10)

u \ v_0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	3	139	331	195	50	14	0	0	0	0	0	0	0	0	0
3	0	4	103	435	447	170	28	8	1	0	0	0	0	0	0	0
4	0	0	8	16	23	18	4	1	0	0	0	0	0	0	0	0
5	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Notes: for v_0, Interval 1 = 0 - 0.25, Interval 2 = 0.25 - 0.50, etc.
for u, Interval 1 = 0 - 0.50, Interval 2 = 0.50 - 1.00, etc.

Fig. 35. Two-dimensional frequency for u and v_0 (n=20)

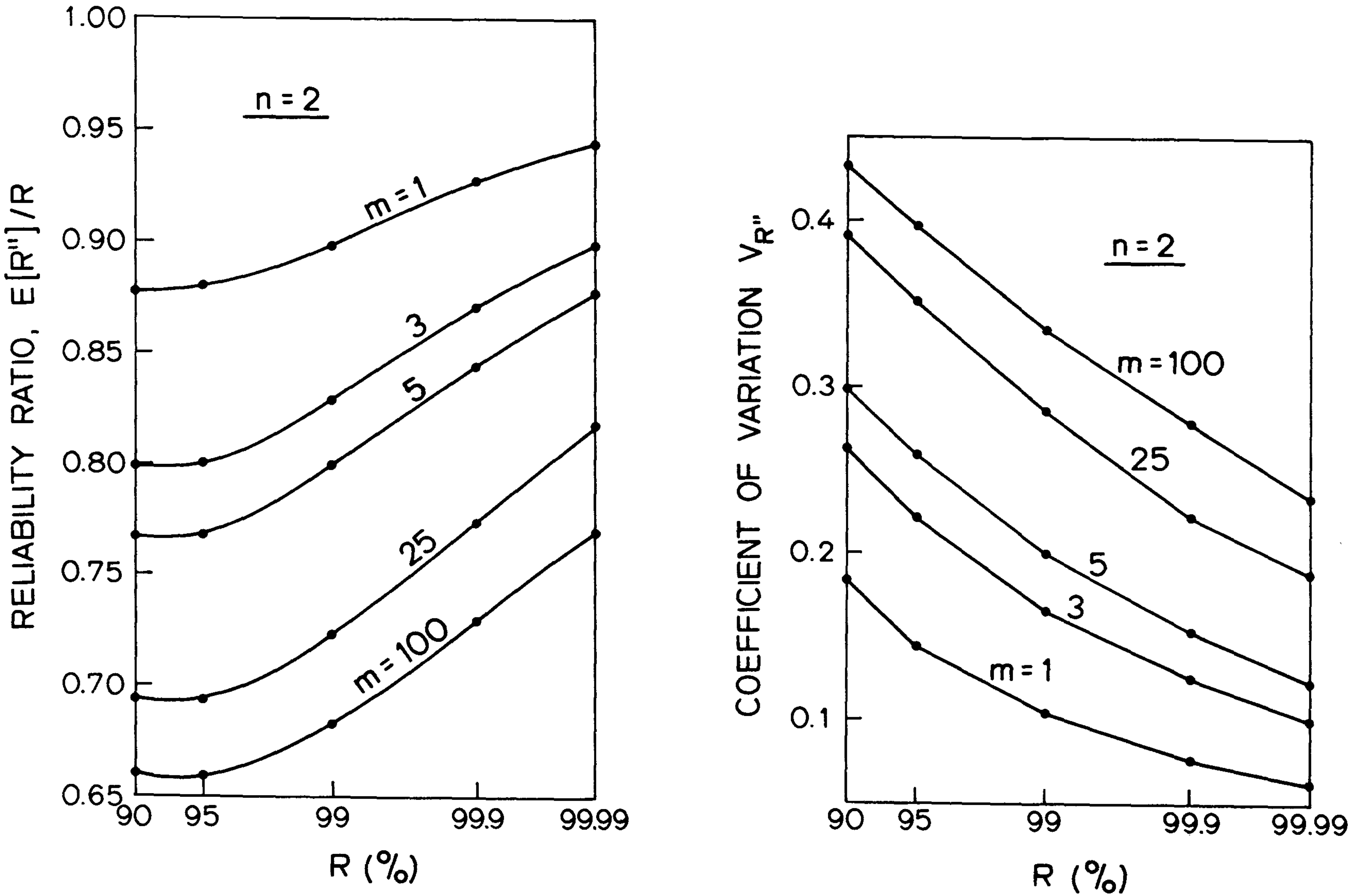

Fig. 36. E[R"]/R and $V_{R''}$ as functions of R (n = 2)

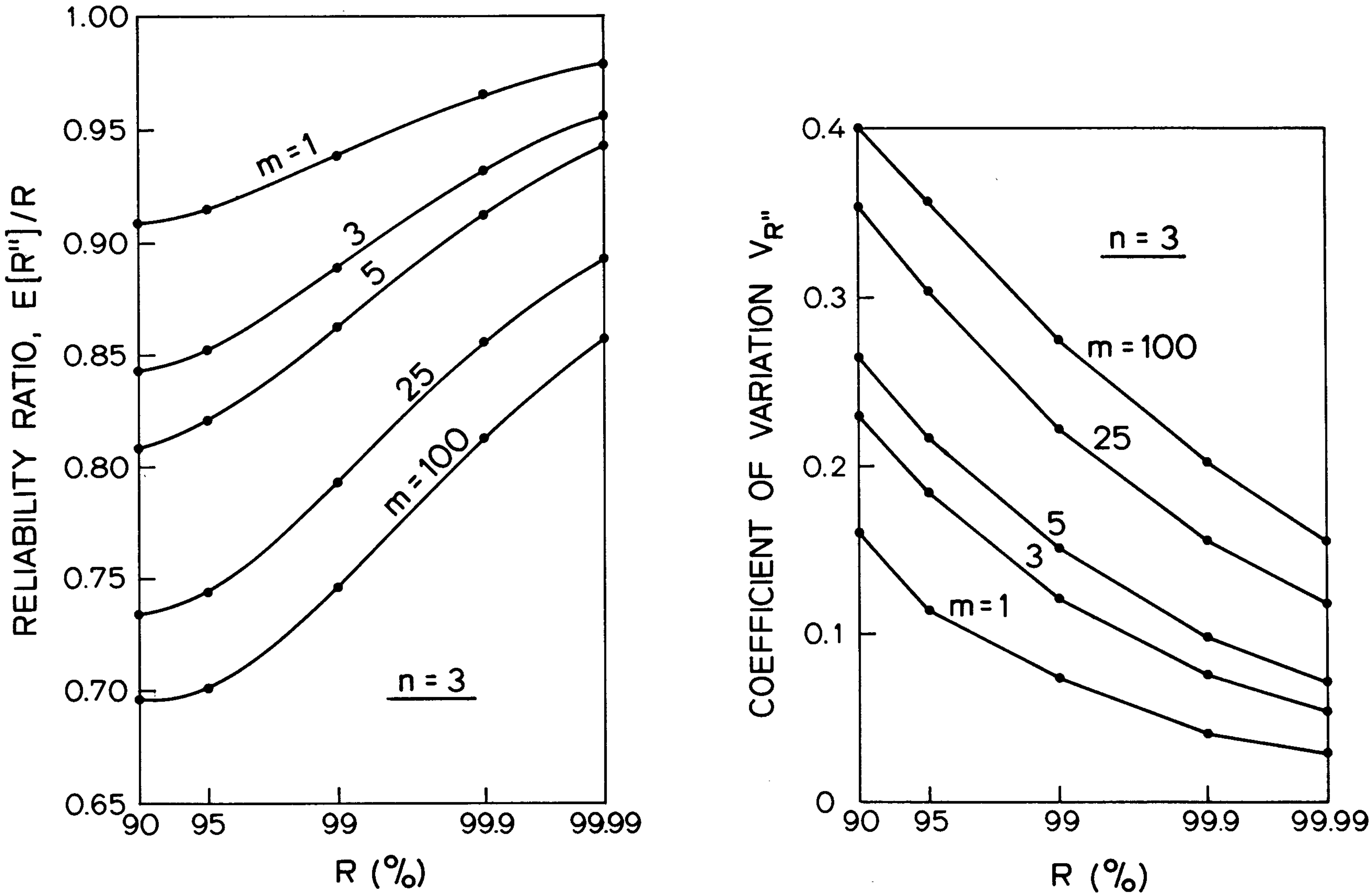

Fig. 37. $E[R'']/R$ and $V_{R''}$ as functions of R (n = 3)

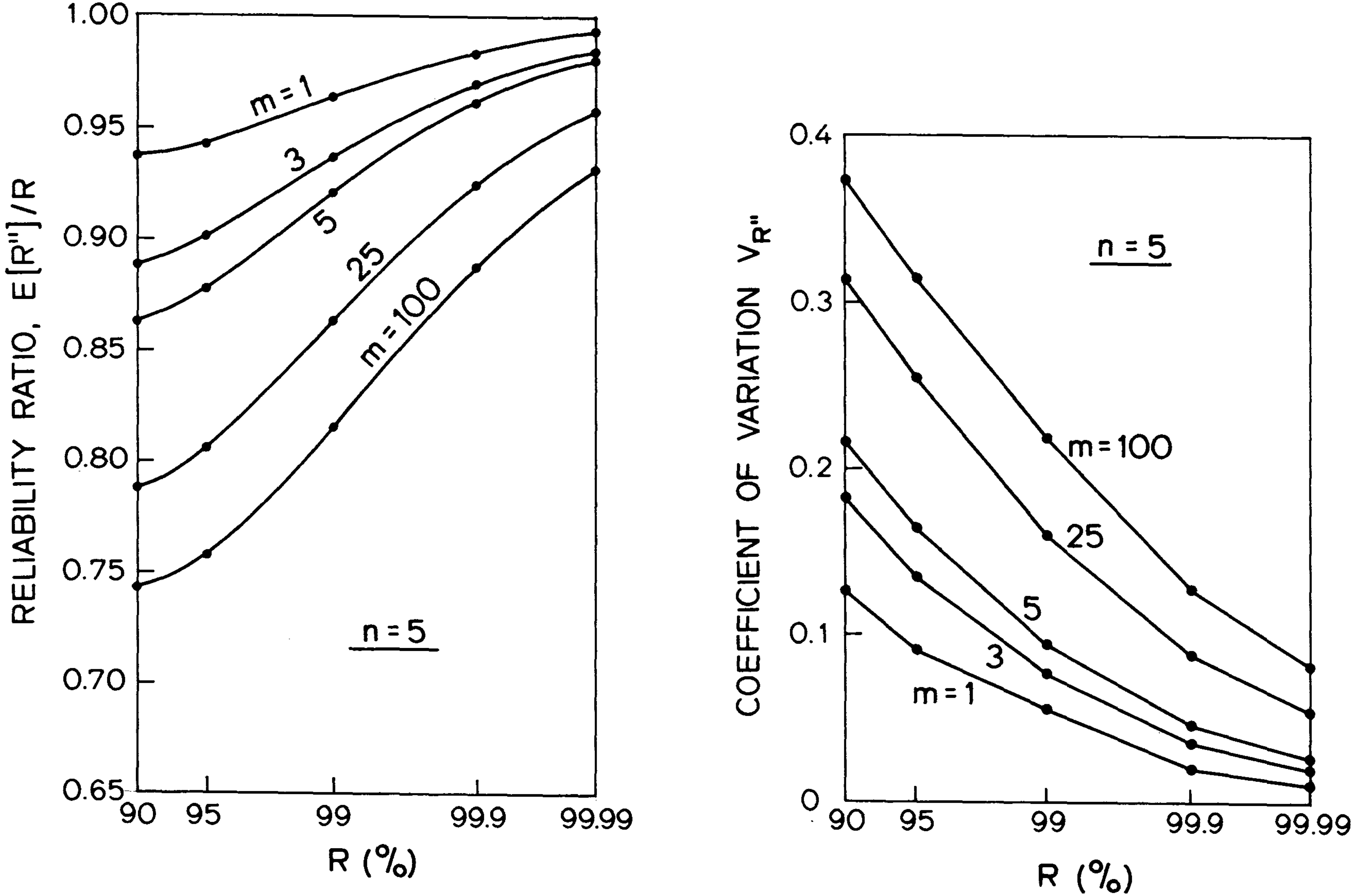

Fig. 38. E[R"]/R and $V_{R''}$ as functions of R (n = 5)

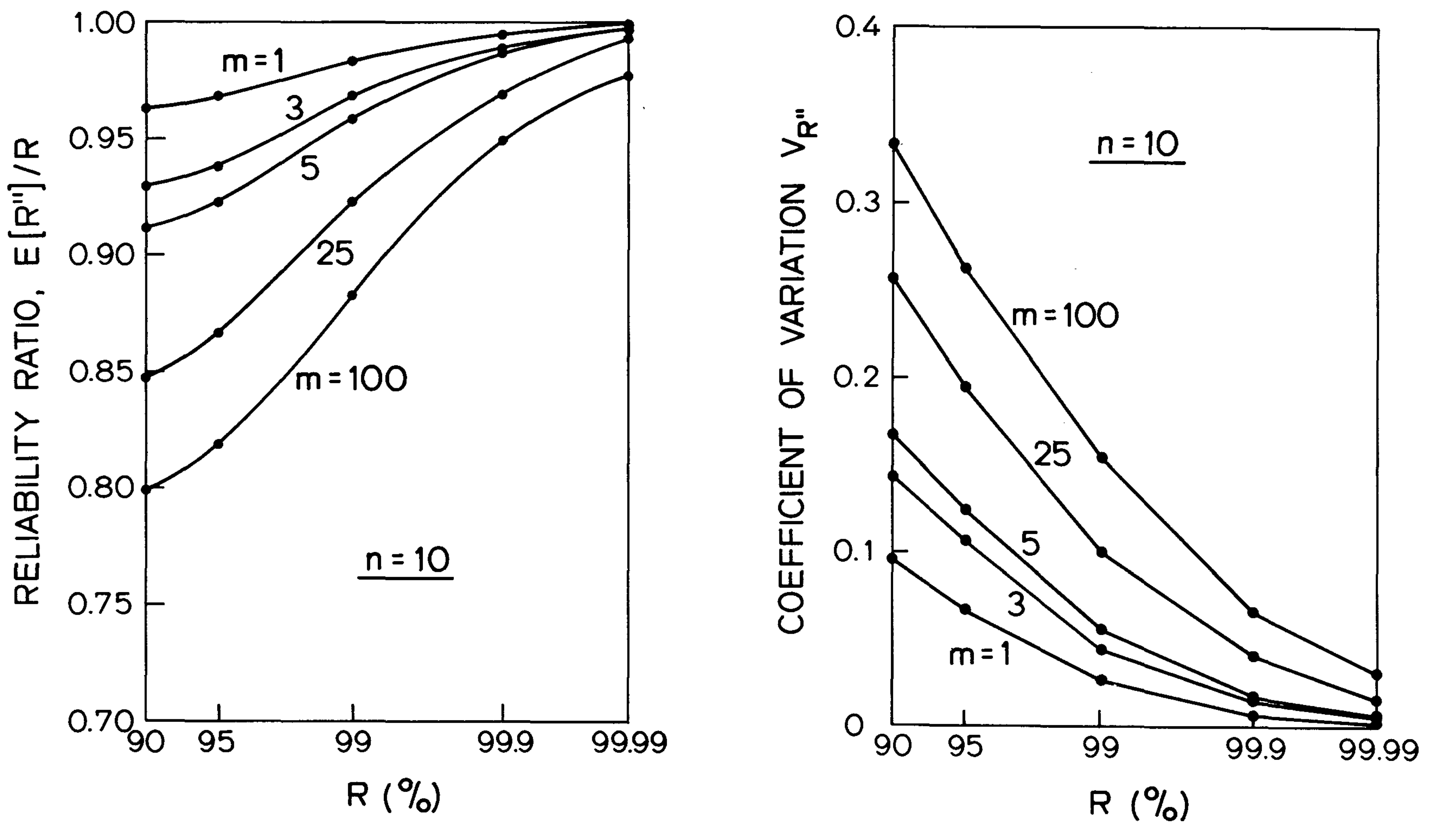

Fig. 39. E[R"]/R and $V_{R"}$ as functions of R (n = 10)

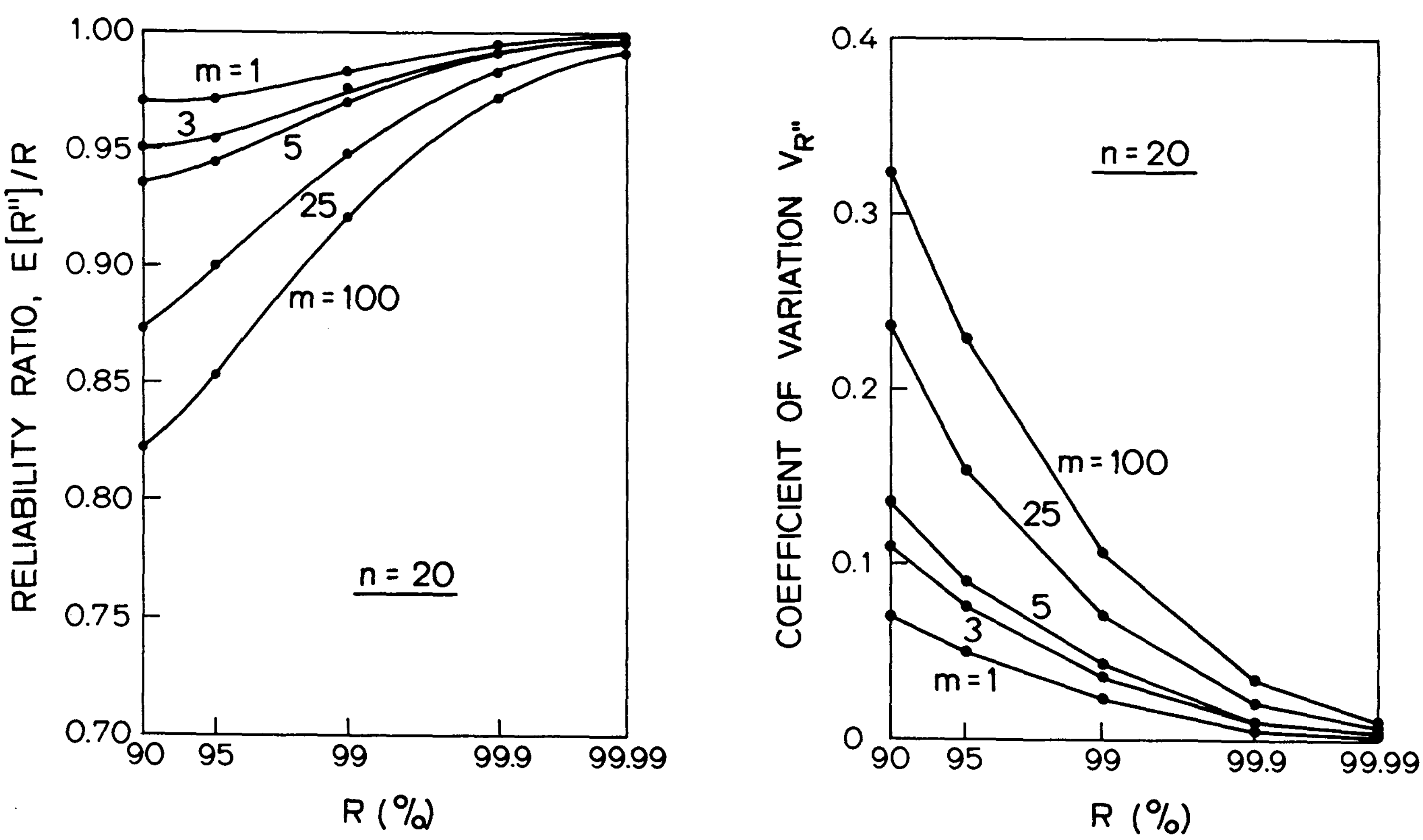

Fig. 40. E[R"]/R and $V_{R''}$ as functions of R (n = 20)

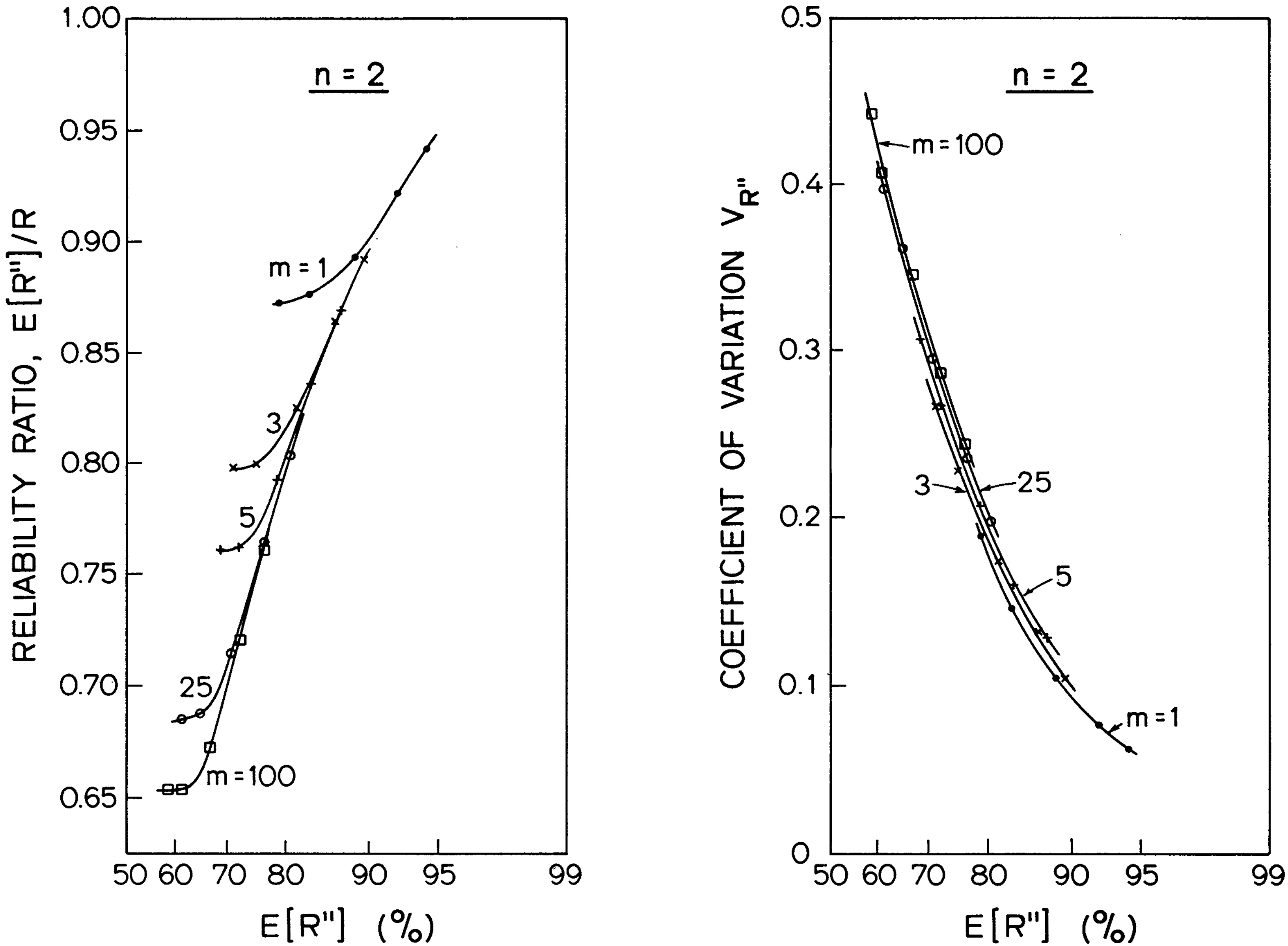

Fig. 41. E[R"]/R and $V_{R''}$ as functions of E[R"] (n=2)

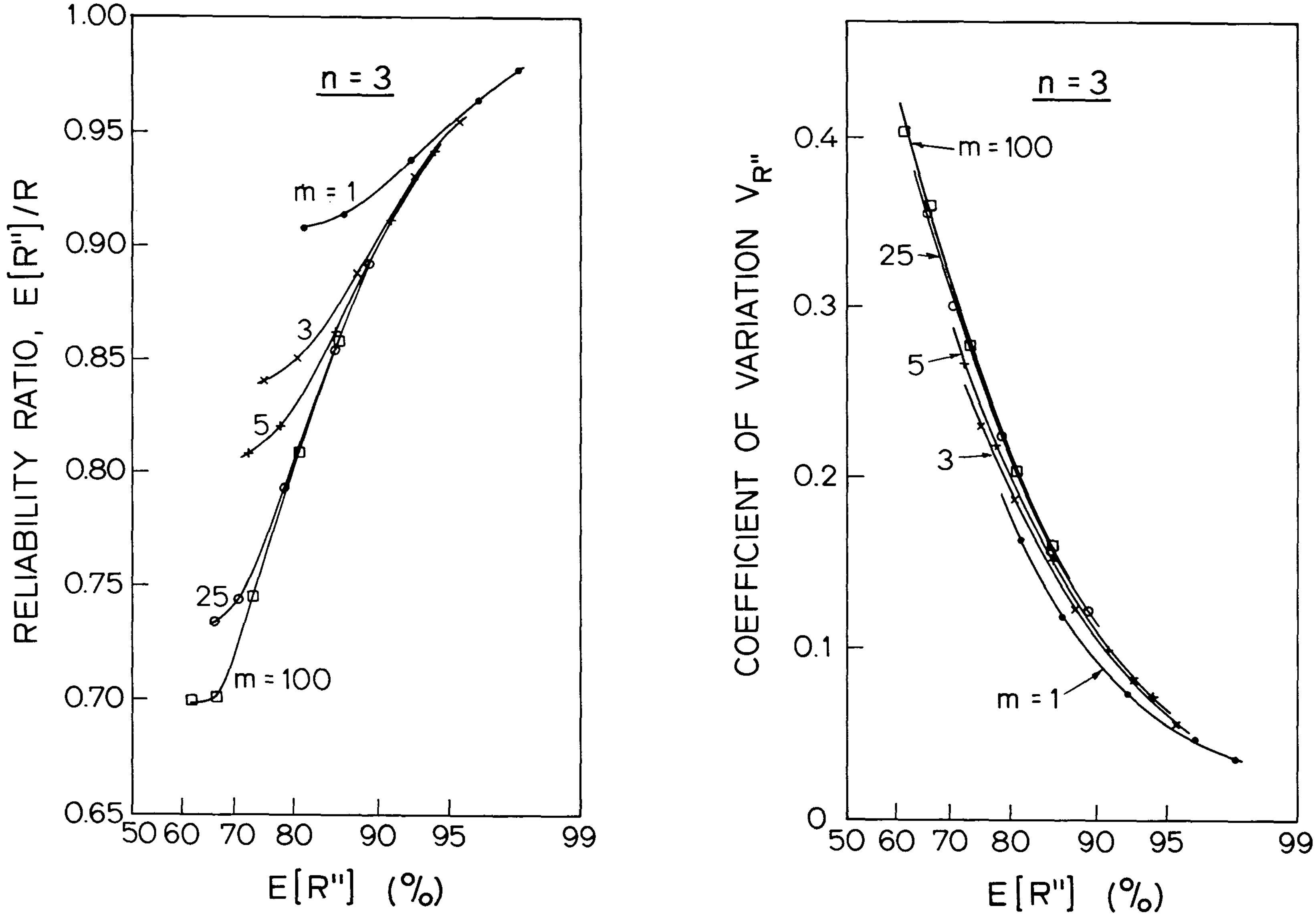

Fig. 42. $E[R'']/R$ and $V_{R''}$ as functions of $E[R'']$ (n=3)

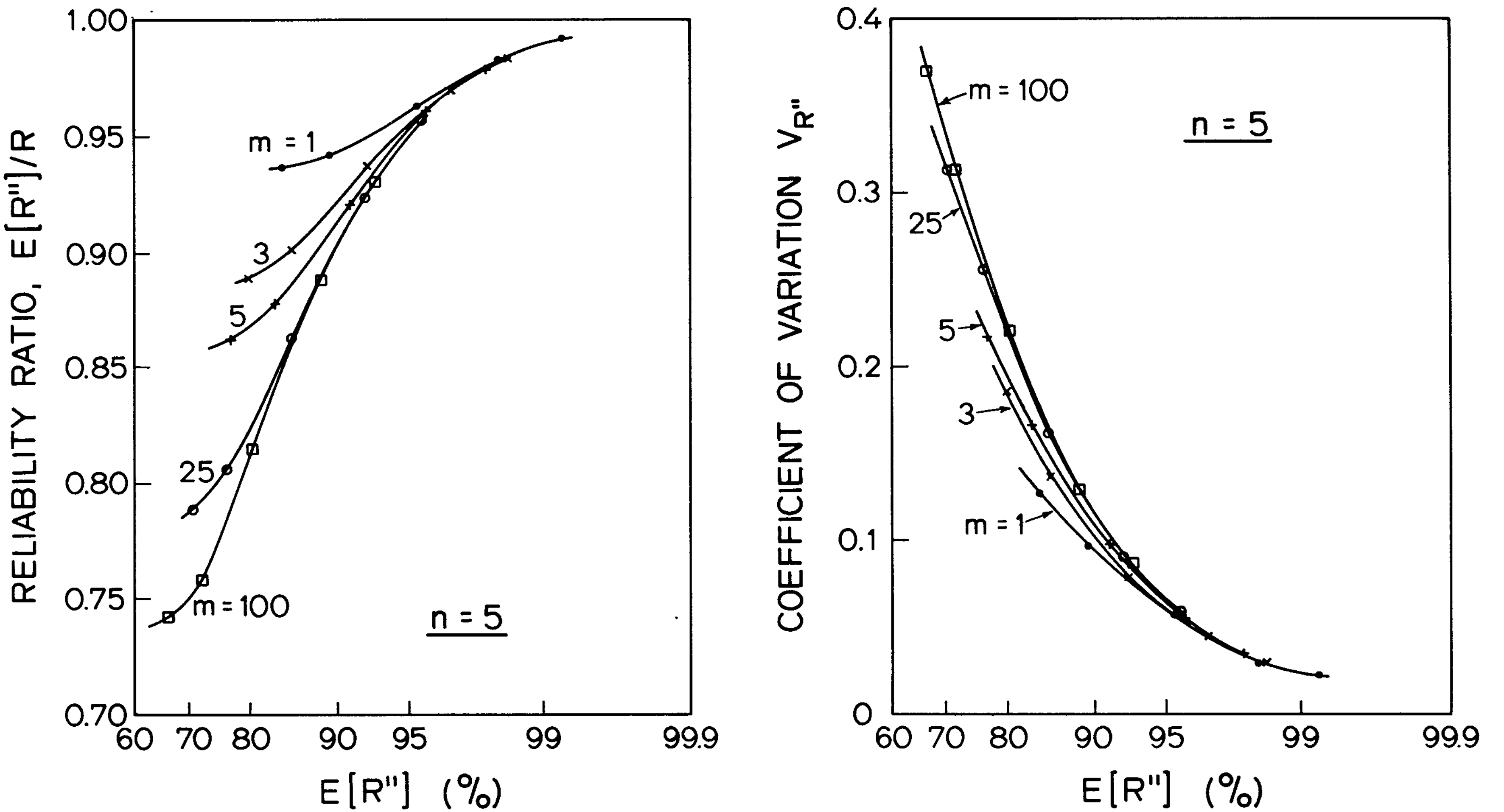

Fig. 43. E[R"]/R and $V_{R''}$ as functions of E[R"] (n=5)

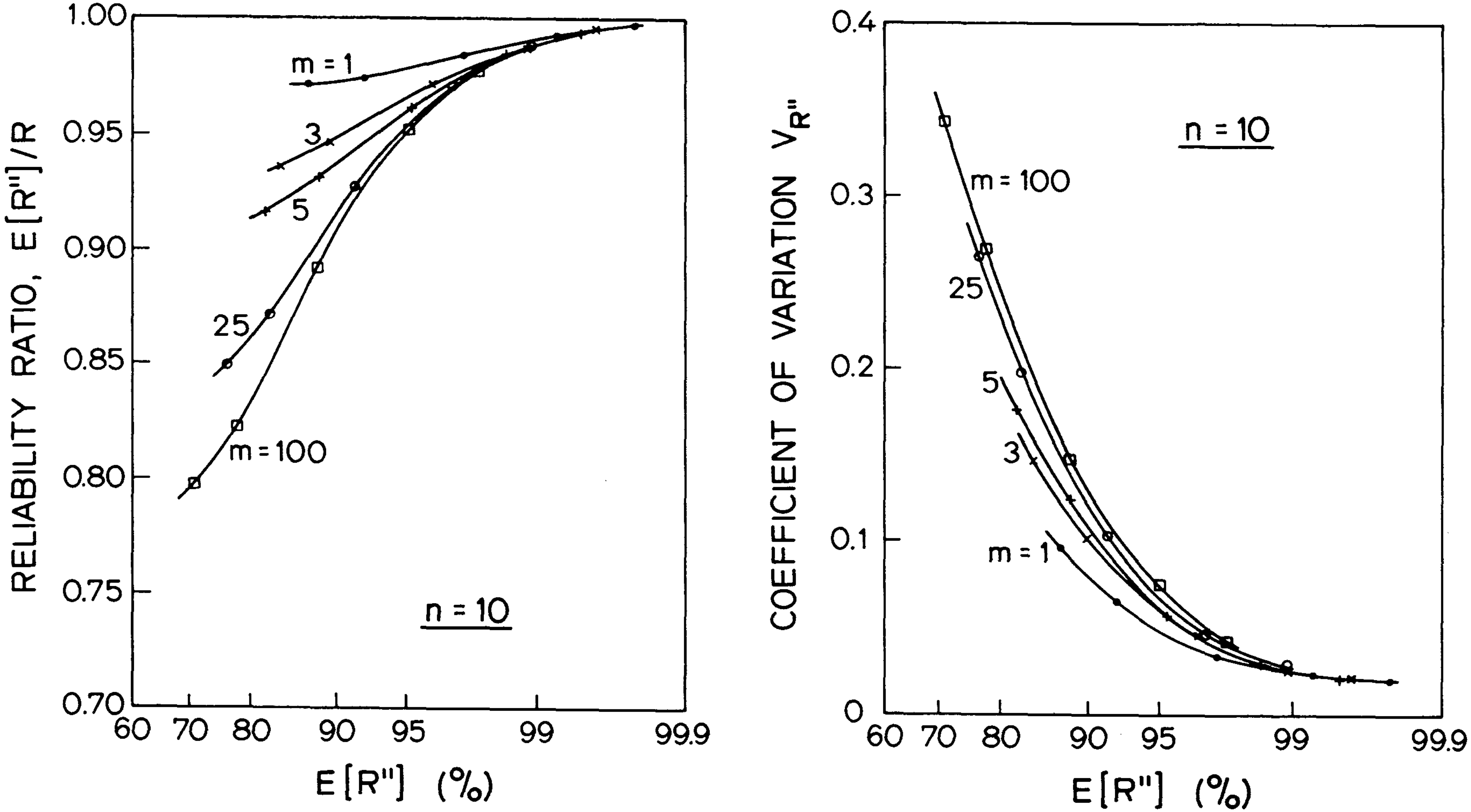

Fig. 44. $E[R'']/R$ and $V_{R''}$ as functions of $E[R'']$ ($n=10$)

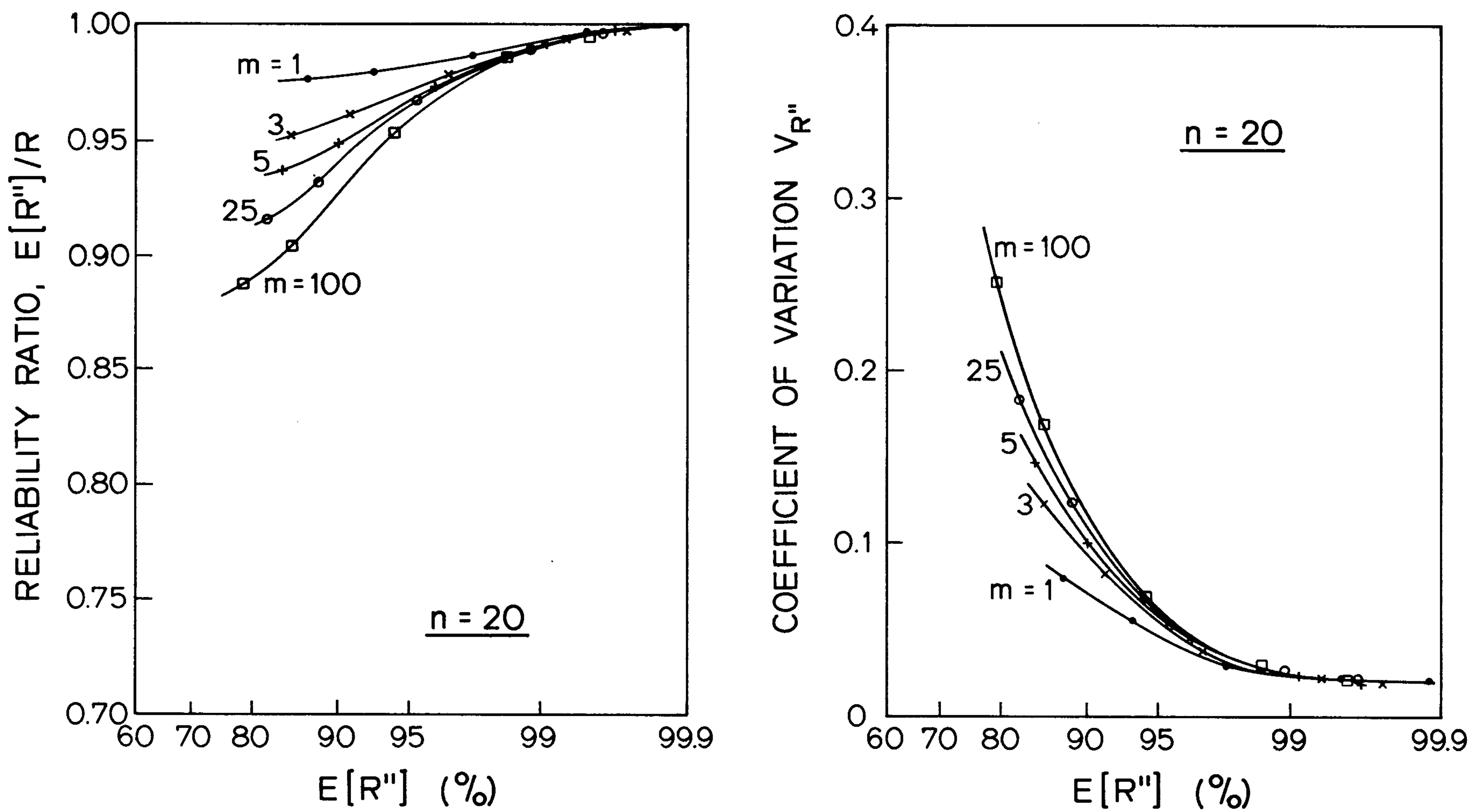

Fig. 45. E[R"]/R and $V_{R''}$ as functions of E[R"] (n=20)

Table 1

Values of E[R'], E[R']/R, $V_{R'}$ and Scatter Factors for

n = 1

R	α	E[R']	E[R']/R	$V_{R'}$	S		
					m=1	m=5	m=25
0.5	2	0.475	0.950	0.488	0.951	2.13	4.76
	3	0.475	0.950	0.488	0.967	1.65	2.83
	4	0.475	0.950	0.488	0.975	1.46	2.18
	5	0.475	0.950	0.488	0.980	1.35	1.87
0.9	2	0.792	0.880	0.253	1.95	4.36	9.76
	3	0.792	0.880	0.253	1.56	2.67	4.57
	4	0.792	0.880	0.253	1.40	2.09	3.12
	5	0.792	0.880	0.253	1.31	1.80	2.49
0.99	2	0.959	0.969	0.0931	4.84	10.8	24.2
	3	0.959	0.969	0.0930	2.86	4.89	8.37
	4	0.959	0.969	0.0930	2.20	3.29	4.92
	5	0.959	0.969	0.0930	1.88	2.59	3.58

Table 2

Values of E[R'], E[R']/R, $V_{R'}$ and Scatter Factors for

n = 5

R	α	E[R']	E[R']/R	$V_{R'}$	S m=1	S m=5	S m=25
0.5	2	0.485	0.970	0.296	1.13	2.53	5.67
	3	0.485	0.970	0.296	1.09	1.86	3.18
	4	0.485	0.970	0.296	1.06	1.59	2.38
	5	0.485	0.970	0.296	1.05	1.45	2.00
0.9	2	0.881	0.979	0.0661	2.79	6.23	13.9
	3	0.881	0.979	0.0661	1.98	3.39	5.79
	4	0.881	0.979	0.0661	1.67	2.50	3.73
	5	0.881	0.979	0.0661	1.51	2.08	2.87
0.99	2	0.988	0.998	0.00721	8.93	20.0	44.7
	3	0.988	0.998	0.00717	4.31	7.36	12.60
	4	0.988	0.998	0.00717	2.99	4.47	6.68
	5	0.988	0.998	0.00717	2.40	3.31	4.57

Table 3

Values of E[R'], E[R']/R , $V_{R'}$ and Scatter Factors for

n = 10

R	α	E[R']	E[R']/R	$V_{R'}$	S m=1	S m=5	S m=25
0.5	2	0.491	0.982	0.216	1.16	2.60	5.82
	3	0.491	0.982	0.216	1.11	1.89	3.24
	4	0.491	0.982	0.216	1.08	1.61	2.41
	5	0.491	0.982	0.216	1.06	1.47	2.02
0.9	2	0.891	0.990	0.0396	2.93	6.56	14.7
	3	0.891	0.990	0.0396	2.05	3.50	5.99
	4	0.891	0.990	0.0396	1.71	2.56	3.83
	5	0.891	0.990	0.0396	1.54	2.12	2.93
0.99	2	0.989	0.999	0.00413	9.47	21.2	47.3
	3	0.989	0.999	0.00406	4.47	7.65	13.1
	4	0.989	0.999	0.00407	3.08	4.60	6.88
	5	0.989	0.999	0.00418	2.46	3.39	4.68

Table 4

Values of E[R"], E[R"]/R, $V_{R''}$ and Q* for n = 2.

R	m	E[R"]	E[R"]/R	$V_{R''}$	Q*
0.9	1	0.789	0.876	0.184	5.46×10
	3	0.720	0.799	0.262	6.08×10^{2}
	5	0.690	0.767	0.300	1.90×10^{3}
	25	0.624	0.693	0.392	7.72×10^{4}
	100	0.594	0.660	0.435	2.07×10^{6}
0.95	1	0.835	0.879	0.144	2.07×10^{2}
	3	0.760	0.800	0.223	1.83×10^{3}
	5	0.730	0.768	0.260	5.73×10^{3}
	25	0.657	0.691	0.353	2.54×10^{5}
	100	0.622	0.655	0.399	8.96×10^{6}
0.99	1	0.888	0.897	0.103	4.20×10^{3}
	3	0.821	0.829	0.169	3.35×10^{4}
	5	0.791	0.799	0.201	8.49×10^{4}
	25	0.716	0.723	0.286	2.98×10^{6}
	100	0.675	0.681	0.335	7.81×10^{7}
0.999	1	0.925	0.926	0.075	2.00×10^{5}
	3	0.869	0.870	0.125	1.71×10^{6}
	5	0.843	0.844	0.152	4.55×10^{6}
	25	0.772	0.773	0.228	1.16×10^{8}
	100	0.729	0.730	0.275	2.98×10^{9}

Table 5

Values of E[R"], E[R"]/R, $V_{R'}$, and Q* for n = 3.

R	m	E[R"]	E[R"]/R	$V_{R"}$	Q*
0.9	1	0.817	0.908	0.162	1.68×10
	3	0.756	0.840	0.230	7.71×10
	5	0.728	0.809	0.265	1.62×10^2
	25	0.661	0.735	0.354	1.50×10^3
	100	0.626	0.696	0.402	1.09×10^4
0.95	1	0.868	0.914	0.120	3.88×10
	3	0.808	0.850	0.185	1.79×10^2
	5	0.780	0.820	0.218	3.35×10^2
	25	0.707	0.744	0.305	3.55×10^3
	100	0.666	0.702	0.357	2.29×10^4
0.99	1	0.928	0.937	0.074	2.53×10^2
	3	0.880	0.888	0.121	1.02×10^3
	5	0.854	0.863	0.149	1.97×10^3
	25	0.785	0.793	0.222	2.31×10^4
	100	0.738	0.746	0.277	1.68×10^5
0.999	1	0.964	0.965	0.047	4.96×10^3
	3	0.930	0.931	0.077	1.59×10^4
	5	0.911	0.912	0.097	2.78×10^4
	25	0.854	0.855	0.156	2.29×10^5
	100	0.809	0.810	0.202	2.31×10^6

Table 6

Values of E[R"], E[R"]/R. $V_{R''}$ and Q* for n = 5

R	m	E[R"]	E[R"]/R	$V_{R''}$	Q*
0.9	1	0.844	0.937	0.126	1.31×10
	3	0.801	0.890	0.183	4.51×10
	5	0.777	0.863	0.217	7.45×10
	25	0.710	0.789	0.313	4.61×10^2
	100	0.669	0.743	0.370	2.76×10^3
0.95	1	0.895	0.942	0.092	2.62×10
	3	0.857	0.902	0.136	8.59×10
	5	0.835	0.879	0.165	1.44×10^2
	25	0.766	0.806	0.256	8.74×10^2
	100	0.721	0.759	0.316	4.86×10^3
0.99	1	0.955	0.964	0.046	1.33×10^2
	3	0.927	0.937	0.077	3.62×10^2
	5	0.912	0.922	0.095	7.10×10^2
	25	0.856	0.865	0.160	4.25×10^3
	100	0.808	0.816	0.220	1.96×10^4
0.999	1	0.983	0.984	0.020	1.30×10^3
	3	0.970	0.971	0.036	3.92×10^3
	5	0.960	0.961	0.048	8.02×10^3
	25	0.924	0.925	0.088	3.63×10^4
	100	0.887	0.888	0.130	1.66×10^5

Table 7

Values of E[R"], E[R"]/R, $V_{R''}$ and Q* for n = 10.

R	m	E[R"]	E[R"]/R	$V_{R''}$	Q*
0.9	1	0.867	0.963	0.096	9.52
	3	0.836	0.928	0.142	2.77×10
	5	0.819	0.910	0.166	4.73×10
	25	0.762	0.847	0.258	2.34×10^2
	100	0.718	0.798	0.334	9.05×10^2
0.95	1	0.920	0.968	0.067	1.68×10
	3	0.892	0.939	0.106	5.26×10
	5	0.877	0.923	0.125	8.83×10
	25	0.823	0.867	0.196	4.35×10^2
	100	0.778	0.819	0.264	1.70×10^3
0.99	1	0.973	0.983	0.026	9.78×10
	3	0.958	0.968	0.045	2.25×10^2
	5	0.949	0.959	0.057	2.96×10^2
	25	0.914	0.923	0.102	1.58×10^3
	100	0.875	0.884	0.154	6.67×10^3
0.999	1	0.993	0.994	0.008	4.98×10^2
	3	0.988	0.989	0.014	1.34×10^3
	5	0.985	0.986	0.018	2.19×10^3
	25	0.968	0.969	0.040	8.39×10^3
	100	0.948	0.949	0.065	3.86×10^4

Table 8

Values of E[R"], E[R"]/R, $V_{R''}$ and Q* for n = 20.

R	m	E[R"]	E[R"]/R	$V_{R''}$	Q*
0.9	1	0.875	0.972	0.069	9.17
	3	0.856	0.951	0.108	2.58×10
	5	0.842	0.935	0.134	4.05×10
	25	0.786	0.873	0.233	1.92×10^2
	100	0.739	0.822	0.324	7.28×10^2
0.95	1	0.921	0.970	0.051	1.87×10
	3	0.906	0.954	0.075	5.19×10
	5	0.897	0.945	0.090	7.30×10
	25	0.856	0.901	0.155	3.20×10^2
	100	0.810	0.853	0.230	1.12×10^3
0.99	1	0.974	0.984	0.025	6.71×10
	3	0.967	0.976	0.035	1.77×10^2
	5	0.961	0.971	0.042	2.84×10^2
	25	0.940	0.949	0.070	1.08×10^3
	100	0.912	0.921	0.107	3.58×10^3
0.999	1	0.994	0.995	0.007	6.62×10^2
	3	0.991	0.992	0.011	2.27×10^3
	5	0.989	0.990	0.013	2.39×10^3
	25	0.981	0.982	0.022	7.27×10^3
	100	0.971	0.972	0.035	2.81×10^4

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